Series solutions and second-order equations

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1. Consider the following initial value problem.

$$\begin{aligned} y'(t) &= 2t \cdot y(t) \\ y(0) &= 3 \end{aligned}$$

   (a) Solve this initial value problem using separation of variables.

   (b) Solve this initial value problem by finding the Taylor series for $y(t)$ directly.

2. Use series to solve the following initial value problem.

$$\begin{aligned} y'(x) &= x + y(x) \\ y(0) &= 2 \end{aligned}$$

3. Find a degree 3 Taylor approximation to the solution of the following initial value problem.

$$\begin{aligned} y' &= 2xy + e^x \\ y(0) &= 1 \end{aligned}$$

Definition 0.1. A *second order differential equation* is a differential equation involving a function and its first two derivatives. An *initial value problem* for a second order differential equation consists of a differential equation plus two initial conditions, specifying the function and its first derivative at a point.

4. Solve the following initial value problem.

$$\begin{aligned} y''(t) &= -9.8 \\ y(0) &= 10 \\ y'(0) &= 4 \end{aligned}$$

Also give a physical interpretation for this initial value problem.

5. Solve the following initial value problem, expressing the solution function as a power series.

$$\begin{aligned} y''(t) &= y(t) \\ y(0) &= 1 \\ y'(0) &= 2 \end{aligned}$$

6. Solve the following initial value problem, expressing the solution function as a power series.

$$\begin{aligned} y''(t) &= -y(t) \\ y(0) &= 1 \\ y'(0) &= 2 \end{aligned}$$

Give a physical interpretation for this situation.