Frequently asked questions for Math 1A

December 6, 2013

This is an **unofficial** list, compiled from questions that I have received in class or during office hours. It reflects what I think will be most helpful to you, but not necessarily official course policy.

1 What facts should I memorize?

In general, my advice on memorizing is: **make your own list as your study.** Every time you encounter a practice problem where you wish you knew a certain fact, write that fact down on a list. By the time you're done studying, you'll have a list of the specific facts that are helpful to you, and you can then spend some time memorizing the items on that list. Different people benefit from memorizing more facts or fewer facts.

That said, here's a rough list of the facts that I think you should **either memorize or be able to re-derive in a few seconds.** I don't claim that this list is exhaustive, and I may add to it as the week goes on.

1. The shapes of the graphs of the following basic functions.

   $$e^x,\ e^{-x},\ \sin x,\ \cos x,\ \tan x,\ x^2,\ \sqrt{x},\ \ln x,\ \arcsin x,\ \arccos x,\ \arctan x$$

   Including especially: the **domain** of each function, where each function is **positive or negative**, where each function is **increasing or decreasing,** and any **asymptotes.**

2. The derivatives of the following functions.

   - e^x, and a^x for positive constants a
   - $\sin x$, $\cos x$, $\tan x$, $\sec x$
   - x^n, especially $\sqrt{x}$ and $\frac{1}{x}$
   - $\ln x$.
   - $\arcsin x$, $\arctan x$

3. The (indefinite) integrals of the following functions.

   - e^x, and a^x for positive constants a
   - $\sin x$, $\cos x$, $\sec^2 x$, $\sec x \tan x$
   - x^n, especially $\sqrt{x}$, $\frac{1}{\sqrt{x}}$, and $\frac{1}{x}$
   - $\frac{1}{1+x^2}$, $\frac{1}{\sqrt{1-x^2}}$

4. The values of sine, cosine, and tangent at the following angles: $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, as well as the corresponding angles in the other quadrants (such as $\frac{2\pi}{3}$, $\frac{5\pi}{6}$, etc.). The corresponding values of the inverse trigonometric functions (e.g. $\arctan 1 = \frac{\pi}{4}$).

5. The fact that the following forms are indeterminate, and the typical techniques for "resolving" them.

   $$\frac{0}{0},\ \frac{\infty}{\infty},\ 0\cdot\infty,\ \infty-\infty,\ 1^\infty,\ 0^0,\ 0^\infty$$

2 How should I decide whether to integrate by parts or with substitution?

I recommend making a list as you study of the problems where you were most confused as to which technique to use. Sort them by which technique works (by reading the solution if necessary), make a note to yourself about *why* one method worked rather than the other, and then study this list later to look for trends.

Also remember that you may sometimes need to use both integration by parts and substitution in one problem. In these cases, the challenge is choosing which to apply first.

That said, here a couple trends that you may find helpful.

1. Is the integrand highly "nested" (lots of compositions of functions)? If so, you probably want to **substitute**.
   Examples: $\int xe^{x^2}dx$, $\int \frac{\cos(\ln x)}{x}$ $\int \sqrt{x}\cos(\sqrt{x})dx$ all have compositions of functions in them; in each case the first step is to substitute for the inner functions (here: $u = x^2$, $u = \ln x$, $u = \sqrt{x}$ respectively).

2. Is the integrand a fraction with a fairly complex denominator? You may want to **substitute** to simplify the denominator (or even replace the whole thing by u).
   Examples: $\int \frac{e^x+\cos x}{e^x+\sin x}dx$, $\int \frac{x+1}{\sqrt[3]{3x^2+6x+5}}dx$

3. Is the integrand a product of two (relatively) simple functions? You probably want to use **parts**.
   Examples: $\int xe^x dx$, $\int x\ln x dx$, $\int x^2\cos x dx$.

4. Is the integrand a single function that you know how to differentiate, but not how to integrate? You may want to try **parts**, where you just take u to be 1.
   Examples: $\int \ln x dx$, $\int \arctan x dx$.

5. Not sure? Try to do both, and see which one succeeds.

3 If I substitute: how do I choose u?

As usual, this comes with practice. I suggest keeping a list of the problems you most struggled with and reviewing it. But here are a couple suggestions that are often helpful.

1. Take u to be the "innermost function."
   Examples: For $\int xe^{x^2}dx$, take $u = x^2$. For $\int \frac{\cos(\ln x)}{x}dx$, take $u = \ln x$.

2. Think in "function-derivative pairs." To substitute successfully, you'll need to spot the derivative of u in the integrand, and express the rest in terms of u.
   Example: For $\int \sin^7 x\cos x dx$, the $\cos x$ is the derivative of $u = \sin x$, and the rest can be expressed in $\sin x$.

3. Try substituting the whole denominator of a fraction.
   Example: For $\int \frac{e^x+\cos x}{e^x+\sin x}dx$, take $u = e^x + \sin x$.

4. For integrands with lots of trigonometric functions, try taking u to be $\sin x$, $\cos x$, or $\tan x$. You may often need to try both sine and cosine to see which one works better.

4 If I integrate by parts: how should I choose u and v?

Once again, this comes with practice. Keep a list of problems that were difficult for you, and review it periodically to try to think of how you'd think of the right choices of u and v.

Here's one method that is successful remarkably often: look at your integrand, and take your u to be the first function you see from this list:

1. First priority for u: Logarithms or inverse trig functions (these become simpler when you differentiate them).
2. Second priority for u: "algebraic" functions like x^n, $\frac{1}{x}$, $\sqrt{x}$, etc.
3. Lowest priority for u: exponential and trigonometric functions. (These keep basically their same form when you integrate them; they don't get any more complex).

One way to remember this order is with the mnemonic "LIATE" (logarithmic-inverse trig-algebraic-trigonometric-exponential). Thanks to Luisa for telling me about this mnemonic in class!

The reason for these priorities is that you want $\int vdu$ to be an easier integral than $\int udv$, so you generally want to choose u to be something that will become simpler on differentiation, and you don't want dv to get any more complicated when you integrate it to v.

Examples:

- $\int xe^x\ dx$: take $u = x,\ dv = e^x dx$. (algebraic over exponential)
- $\int x^3 \tan^{-1} x dx$: take $u = \tan^{-1} x, dv = x^3 dx$. (inverse trig over algebraic)
- $\int \ln x dx$; take $u = \ln x,\ dv = dx$. (logarithmic is first priority, even if nothing else is there).

5 When do I change the limits of integration during a computation?

You only change the limits of integration when you substitute, not when integrate by parts.

Examples:

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$$\begin{aligned} \int_1^3 \sqrt{2x+1}dx & \qquad u = 2x+1,\ du = 2dx \\ &= \int_{2*1+1}^{2*3+1} \frac{1}{2}\sqrt{u}\ du \quad \text{(limits changed since we are substituting).} \\ &= \int_3^7 \frac{1}{2}\sqrt{u}du \end{aligned}$$

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$$\begin{aligned} \int_0^\pi x \sin x dx & \qquad u = x,\ dv = \sin x dx \\ & \qquad du = dx,\ v = -\cos x \\ &= [-x\cos x]_0^\pi + \int_0^\pi \cos x dx \quad \text{(limits don't change for int. by parts)} \end{aligned}$$

One way to remember this: if the integral is a "dx integral," then the limits should be values of x. If it changes into a "du integral," you need to replace the limits with values of u instead. When we integrate by parts, we end up with an integral that is still a "dx integral," so the limits don't change.

6 What topics will the exam cover?

All topics from the whole semester are fair game. There will be greater emphasis on the last third of the material, since it has not yet been covered on any exams.

7 How much work should I show?

A good rule of thumb is: write your answers as if you are writing for a peer in the class. You should show enough work that another student could read your answer and understand exactly what you are doing.

Here are a couple things that are generally smart to always write down.

1. When you substitute, write down what u is (and what this makes du).
2. When you use parts, write down what u and v are (as well as du and dv).
3. When you take a limit with l'Hôpital, make a short note to this effect (e.g. write "l'Hôp" next to your new limit) so we know that's what you've done.

Here are a couple things you can usually safely leave out of your shown work.

1. Easy substitutions, such as $\int e^{2x}dx = \frac{1}{2}e^{2x} + C$, or $\int \cos(4x+3)dx = \frac{1}{4}\sin(4x+3) + C$. It doesn't hurt to write what u is (in these cases: $u = 2x$ and $u = 4x + 3$, respectively).
2. Derivations of integrals, derivatives, or limits you know off the top of your head. For example, if you have memorized $\int \tan x dx = \ln|\sec x| + C$, you don't need to compute it from scratch.

8 Should I simplify my answer?

No, you will not need to simplify your answer for full points, unless the problem *specifically* requests the answer in a certain form.

However, it is often smart to simplify your answer, since it may make it easier for you to check that it makes sense. In multi-part problems, a simplified answer may be much simpler to use in a later part. It is also courteous to your grader. But if you are short on time, don't spend much effort on it.