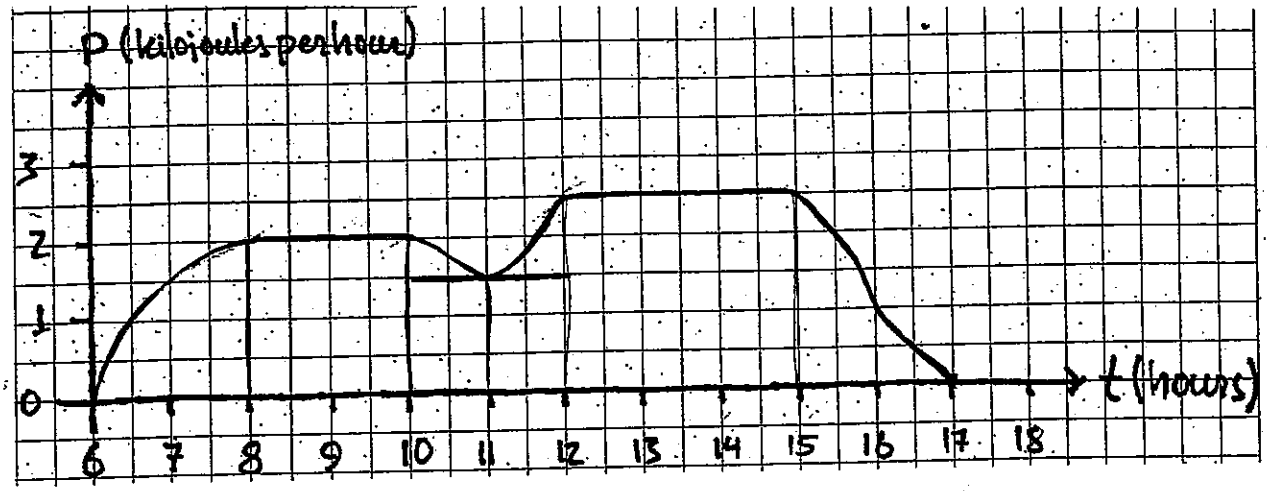
§5.1
§5.2

Worksheet for 11/12/13

§5.4 Thursday

① The following graph shows the power output of a solar panel (in kilojoules per hour) over the course of a day. Here, t is the number of hours since midnight.



a) How much energy (in kJ) does the panel produce between 12pm and 3pm?

$2.5 \frac{kJ}{hr} \cdot 3hr = \boxed{7.5\ kJ}$

b) Between 8am and 10am?

$2 \frac{kJ}{hr} \cdot 2hr = \boxed{4\ kJ}$

c) Between 10am and 12pm?

roughly:

$1.5 \cdot 2 + 0.75 + 0.5 = \boxed{3.75\ kJ}$ (approximate)

d) Between 6:30am and 4:30pm?

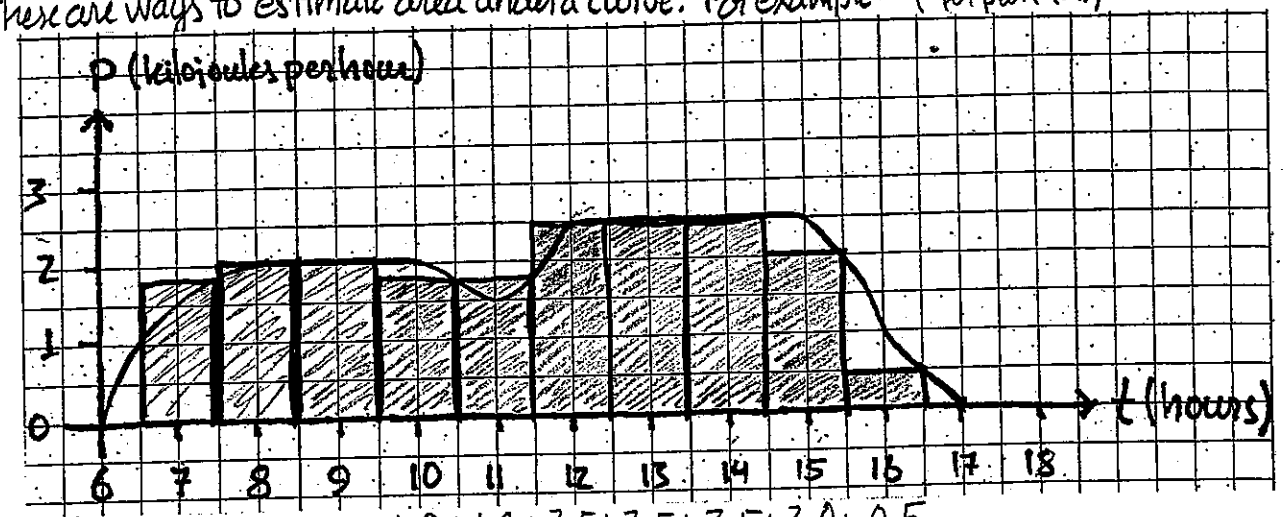
See back side for some estimates.

Close to 19.5 kJ.

R_n means: "right-hand sum with n subdivisions"

L_n means "left-hand sum with n subdivisions".

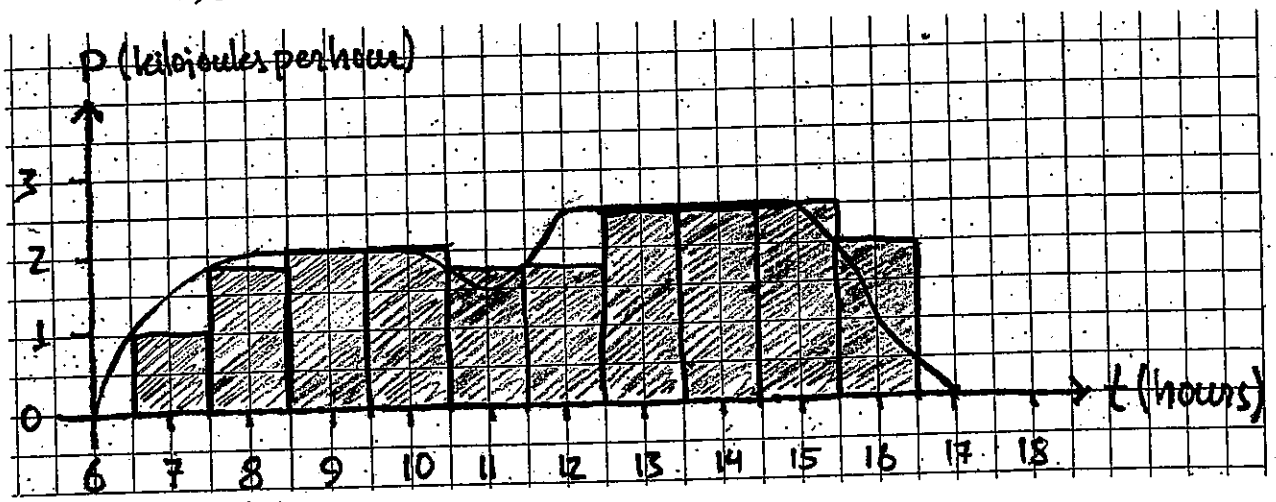
These are ways to estimate area under a curve. For example: (for part (d)):



$R_{10} = 1.8 + 2.0 + 2.0 + 1.8 + 1.8 + 2.5 + 2.5 + 2.5 + 2.0 + 0.5$

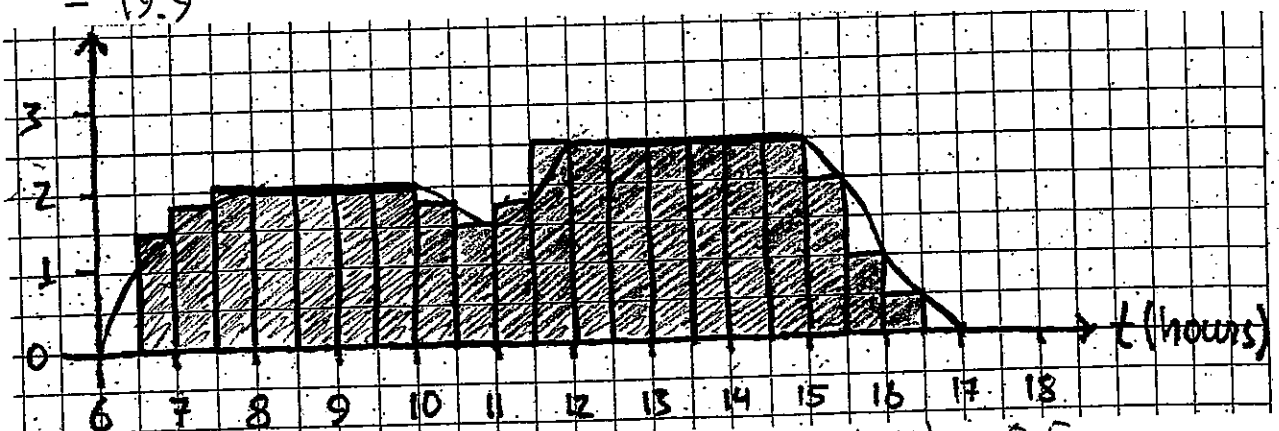
(all rectangles are width 1)

$= 19.4$



$L_{10} = 1.0 + 1.8 + 2.0 + 2.0 + 1.8 + 1.8 + 2.5 + 2.5 + 2.5 + 2.0$

$= 19.9$



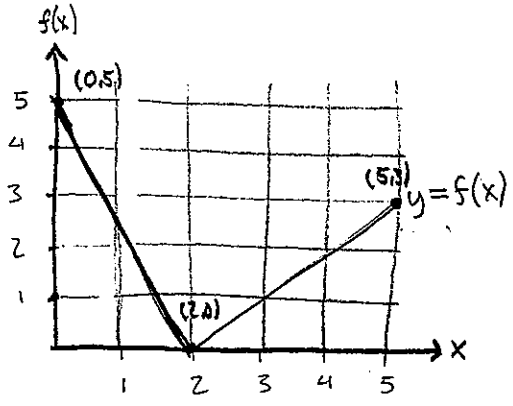
$R_{20} = ((1.5 + 1.8 + 2.0 + 2.0 + 2.0 + 2.0 + 2.0 + 1.8 + 1.5 + 1.8 + 2.5 + 2.5 + 2.5 + 2.5 + 2.5 + 2.5 + 2.0 + 1.0 + 0.5) \cdot 0.5$

(0.5 = width of each rectangle)

$= 19.7$

Part II

(2) Consider the following piecewise-linear function.



a) Evaluate $\int_0^2 f(x)dx$.

$\frac{1}{2} \cdot 5 \cdot 2 = \boxed{5}$

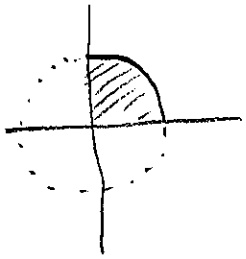
b) Evaluate $\int_2^5 f(x)dx$

$\frac{1}{2} \cdot 3 \cdot 3 = \boxed{4.5}$

c) Evaluate $\int_0^5 f(x)dx$

$5 + 4.5 = \boxed{9.5}$

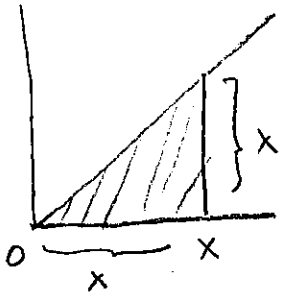
(3) Evaluate $\int_0^1 \sqrt{1-x^2}\,dx$ (by drawing a picture)



$\frac{1}{4}$ of a circle of radius 1

$= \boxed{\pi/4}$

(4) Evaluate $\int_0^x t\,dt$ as a function of x



$\frac{1}{2} x \cdot x = \boxed{\frac{1}{2}x^2}$

Tolstoy, War & Peace
Pevear & Volokhonsky translation

Part Three

I

For human reason, absolute continuity of movement is incomprehensible. Man begins to understand the laws of any kind of movement only when he examines the arbitrarily chosen units of that movement. But at the same time it is from this arbitrary division of continuous movement into discrete units that the greater part of human errors proceeds.

A well-known so-called sophism of the ancients posits that Achilles can never overtake a tortoise that is walking ahead of him, even though Achilles walks ten times faster than the tortoise: while Achilles covers the distance that separates him from the tortoise, the tortoise will get ahead of him by one tenth of that distance; Achilles covers that one tenth, the tortoise gets ahead by one hundredth, and so on to infinity. The ancients considered this problem insoluble. The nonsensical conclusion (that Achilles will never overtake the tortoise) resulted only from the fact that discrete units of movement were introduced arbitrarily, while the movement of both Achilles and the tortoise was continuous.

By taking smaller and smaller units of movement, we only approach the solution of the problem, but never reach it. Only by allowing for an infinitesimal quantity and the ascending progression from that up to one tenth, and by taking the sum of that geometrical progression, do we arrive at the solution of the problem. A new branch of mathematics, having attained to the art of dealing with infinitesimal quantities in other, more complex problems of movement as well, now gives answers to questions that used to seem insoluble. calculus

This new branch of mathematics, unknown to the ancients, in examining questions of movement, allows for infinitesimal quantities, that is, such as restore the main condition of movement (absolute continuity), and thereby corrects the inevitable error that human reason cannot help committing when it examines discrete units of movement instead of continuous movement.

The same thing happens in the search for the laws of historical movement.

The movement of mankind, proceeding from a countless number of human wills, occurs continuously.

To comprehend the laws of this movement is the goal of history. But in order to comprehend the laws of the continuous movement of the sum of all individual wills, human reason allows for arbitrary, discrete units. The first method of

history consists in taking an arbitrary series of continuous events and examining it separately from others, whereas there is not and cannot be a beginning to any event, but one event always continuously follows another. The second method consists in examining the actions of one person, a king, a commander, as the sum of individual wills, whereas the sum of individual wills is never expressed in the activity of one historical person.

Historical science in its movement always takes ever smaller units for examination, and in this way strives to approach the truth. But however small the units that history takes, we feel that allowing for a unit that is separate from another, allowing for the *beginning* of some phenomenon, and allowing for the notion that all individual wills are expressed in the actions of one historical person, is false in itself.

Any conclusion of historical science, without the least effort on the part of criticism, falls apart like dust, leaving nothing behind, only as a result of the fact that criticism selects as an object for observation a larger or smaller discrete unit, which it always has the right to do, because any chosen historical unit is always arbitrary.

Only by admitting an infinitesimal unit for observation—a differential of history, that is, the uniform strivings of people—and attaining to the art of integrating them (taking the sums of these infinitesimal quantities) can we hope to comprehend the laws of history.

The first fifteen years of the nineteenth century in Europe present an extraordinary movement of millions of people. People abandon their usual occupations, rush from one side of Europe to the other, plunder, kill each other, triumph and despair, and the whole course of life is altered for several years and presents an intense movement, which initially increases, then weakens. Human reason asks, what was the cause of this movement, or according to what laws did it occur?

Historians, in answer to this question, lay before us the deeds and speeches of several dozen men in one of the buildings in the city of Paris, calling these deeds and speeches by the name of *revolution;* then they give a detailed biography of Napoleon and of some persons sympathetic or hostile to him, tell of the influence of some of these persons on others, and say: here is the origin of this movement, and here are its laws.

But human reason not only refuses to believe in this explanation, but says straight out that this method of explaining is incorrect, because in this explanation a weaker phenomenon is taken as the cause of a stronger one. The sum of individual human wills produced the revolution and Napoleon, and only the sum of those wills endured them and then destroyed them.

"But every time there were conquests, there were conquerors; every time