Worksheet for 11/5/13

Evaluate the following limits:

(1) $\lim_{x \to \infty} \frac{\ln x}{\sqrt[10]{x}}$

(2) $\lim_{x \to 0} \frac{e^x - e^{-x}}{\sin x}$

(3) $\lim_{x \to 0} \frac{e^x + e^{-x}}{\cos x}$

(4) $\lim_{x \to 0} \frac{\sin x}{\tan^{-1} x}$

(5) $\lim_{x \to 1} \frac{\cos\left(\frac{\pi}{2} \cdot x\right)}{\sqrt{x} - 1}$

(6) $\lim_{x \to \infty} \frac{e^x}{x^4}$

(7) $\lim_{x \to \infty} \frac{\ln(\ln x)}{\sqrt{x}}$

(8) $\lim_{x \to 0} \frac{x^3}{x - \tan^{-1} x}$

(9) $\lim_{x \to \pi} \frac{\sin x + x - \pi}{(x - \pi)^3}$

(10) $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$

Part II

(1) (cf: PSet 8, #5 and current PSet #1). Show with examples why the following forms are indeterminate.

a) $0/0$

b) ∞/∞

c) $\infty - \infty$

d) $0 \cdot \infty$

e) 0^0

f) 1^∞ (hint: PSet 9, A6). (many other approaches, too).

Example $0/0$ is indeterminate since

$$\lim_{x \to 0} \frac{2x}{x} = 2 \qquad \text{but} \qquad \lim_{x \to 0} \frac{x}{2x} = \frac{1}{2}$$

even though both have the form $\frac{0}{0}$ if evaluated naively.

More precisely: if $\lim_{x \to c} f(x) = 0$ and $\lim_{x \to c} g(x) = 0$, then

$\lim_{x \to c} \frac{f(x)}{g(x)}$ could still be anything.

(2) Evaluate the following limits. (We will probably return to these on Thursday).

~~a) $\lim_{x \to \infty} (x(\tan^{-1} x$~~

a) $\lim_{x \to \infty} \left[x \cdot \left(\tan^{-1} x - \frac{\pi}{2} \right) \right]$

b) $\lim_{x \to 0^+} (x^x)$

c) $\lim_{x \to 1^+} \left(x^{1/(x-1)} \right)$

d) $\lim_{x \to \infty} \left(\sqrt{x^2+x} - x \right)$

Note One of these is much easier to do <u>without</u> l'hôpital's rule.