Review problems

Some corrections to the original, written in blue.

Part I: fast recall

Diff. rules

(1) Differentiate:

a) x^{27}
b) $\sqrt{x}$
c) $1/x$
d) e^x
e) $\ln x$
f) $\cos x$
g) $\tan x$
h) $\tan^{-1} x$
i) $\ln(\sin x)$
j) $x \cdot e^x$

Linear approx.

(2) Estimate $\sin(0.02)$ w/ linear approximation.

optimisation

(3) Find the ~~maximum and~~ minimum of $f(x) = e^x - x$ on $(-\infty, \infty)$.

(4) Find the maximum and minimum of $f(x) = x^3 - 3x$ on $[0, 2]$.

graphing

(5) Sketch the graph of $f(x) = x^3 - 3x$.

(6) Evaluate $\lim_{x \to -\infty} \frac{e^x + 1}{e^x - 1}$ and $\lim_{x \to \infty} \frac{e^x + 1}{e^x - 1}$.

related rates

(7) Suppose f, g are functions such that $f(t)g(t) = t$ for all t, $f(0) = ~~0~~ 7$, $g(0) = ~~7~~ 0$ and $f'(0) = 1$. Find $g'(0)$.

Part II : longer problems.

(8) (Differentiation) Find derivatives:

a) $\ln(\ln(\ln(x)))$

b) $\dfrac{e^x\sqrt{\sin x}}{(x+1)^7(x-2)^9}$

c) $\sec^{-1}x$ for $x>0$ only.

d) $e^{x^2}\cdot\ln(7x+1)$

(9) (optimization)

a) Find the maximum area of a right triangle w/ hypotenuse of length 1.

b) Find the maximum perimeter of ~~such~~ a right triangle with hypotenuse of length 1.

(10) (optimization)
Find the minimum perimeter of a slice of pie of area $50\ cm^2$.

(11) (linear approximation)

a) Approximate $\tan^{-1}(1.03)$ (answer in terms of π).

b) Find the tangent line to $y^2 = x^3 - x$ at the point $(2, \sqrt{6})$.

(12) (graphing) Sketch the graph of $f(x) = x \cdot e^{-x^2}$ you may assume that $\lim_{x \to \pm\infty} f(x) = 0$ (we haven't covered how to evaluate these limit in detail).

-Find all local extrema, and determine where the function is increasing and decreasing.
-Determine where it is concave up or down.
-Give a rough sketch from this information.

(13) (graphing) Sketch the graph of $f(x) = \dfrac{-3x^2+4x}{x^2-4}$.

-Find all vertical asymptotes, with the limits on both sides.
-Find all horizontal asymptotes.
-Give a rough sketch from this information.

(14) (Related rates)

A certain town has a workforce of 50,000 people, of whom 10% are unemployed. Suppose that the workforce is growing at a rate of 900 people per month, and 400 new jobs are being created per month. What is the rate of change of the unemployment rate (expressed in percent per month)?