1. (Textbook 13.1 through 13.4)
   (This sounds like a lot, but 13.1 is most of the work, and the others are fairly short. You should read the worked examples in Chapter 13 to see some applicable techniques.)

2. (Textbook 13.14)

3. As mentioned in class, $\mathbb{F}_2[t]/(t^3+t^2+1) \cong \mathbb{F}_2[t]/(t^3+t+1)$, and both fields can be referred to as $\mathbb{F}_8$. Exhibit an explicit bijection between these two quotients.

4. Construct a field of $\mathbb{F}_{16}$ order 16. For each of the 16 elements, determine its $\mathbb{F}_2$-conjugates, degree over $\mathbb{F}_2$, and minimal polynomial over $\mathbb{F}_2$ (try to find a way to present this information efficiently). Determine the Galois group of $\mathbb{F}_{16}/\mathbb{F}_2$ and all of its subgroups, and use this to list all subfields. For each subfield, explicitly list the elements.

5. Let p be an *odd* prime, and let $L/\mathbb{F}_p$ be the splitting field of t^4+1, let ζ denote a root of t^4+1 in L, and let $\omega = \zeta + \zeta^{-1}$.

   (a) Prove that $\omega^2 = 2$, and therefore that t^2-2 splits in $\mathbb{F}_p$ if and only if $\omega \in \mathbb{F}_p$. We say that 2 is a *quadratic residue mod p* if t^2-2 splits in $\mathbb{F}_p$.

   (b) Prove that $\omega^p = \begin{cases} \omega & \text{if } p \equiv \pm 1 \mod 8 \\ -\omega & \text{if } p \equiv \pm 3 \mod 8 \end{cases}$.

   (c) Deduce that 2 is a quadratic residue modulo p if and only if $p \equiv \pm 1 \mod 8$.