Math 111 • (Prof. Folsom)

Sigma Notation

We will use the shorthand $\sum$-notation for sums. This is built using the greek letter $\sum$ (sigma). (Think "S" for sums.)

$$\sum_{k=1}^{n} r(k) = r(1) + r(2) + \cdots + r(n-1) + r(n).$$

Here are some examples.
Note that the lower limit of summation doesn't always have to start at 1.
Note also that we don't always use "k" to be the index of summation.

- $\displaystyle\sum_{k=0}^{3} k = 0 + 1 + 2 + 3 = 6$
- $\displaystyle\sum_{k=0}^{3} \frac{k-1}{4} = \frac{0-1}{4} + \frac{1-1}{4} + \frac{2-1}{4} + \frac{3-1}{4} = \frac{1}{2}$
- $\displaystyle\sum_{i=0}^{2} \cos(3i - \pi) = \cos(3 \cdot 0 - \pi) + \cos(3 \cdot 1 - \pi) + \cos(3 \cdot 2 - \pi)$
- $\displaystyle\sum_{j=6}^{8} \sqrt{j^2 + 5} = \sqrt{6^2 + 5} + \sqrt{7^2 + 5} + \sqrt{8^2 + 5}$
- $\displaystyle\sum_{j=1}^{n} \sqrt{j^2 + 5} = \sqrt{1^2 + 5} + \sqrt{2^2 + 5} + \cdots + \sqrt{(n-1)^2 + 5} + \sqrt{n^2 + 5}$
- $\displaystyle\sum_{i=1}^{n} f(x_i)\Delta x = f(x_1)\Delta x + f(x_2)\Delta x + \cdots f(x_{n-1})\Delta x + f(x_n)\Delta x$
- Write $\frac{1}{2} + 1 + \frac{3}{2} + 2 + \frac{5}{2} + 3$ using $\sum$-notation.
  Solution. We see a sum of 6 terms, so we try a sum $\sum_{j=1}^{6}$. This one will work (other answers are possible too)

$$\sum_{j=1}^{6} \frac{j}{2} = \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \frac{6}{2} = \frac{1}{2} + 1 + \frac{3}{2} + 2 + \frac{5}{2} + 3 \checkmark.$$