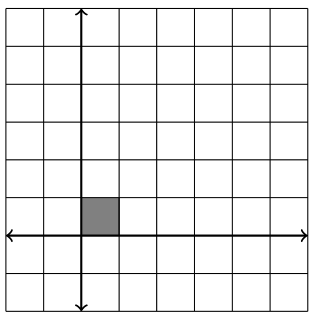
1. [9 points] Solve the following linear system of equations.

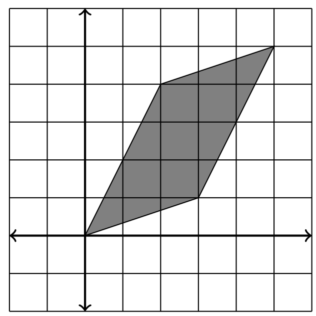
$$\begin{array}{rrrrcl} & 4x_2 & +12x_3 & -7x_4 & = & 8 \\ & x_2 & +3x_3 & -2x_4 & = & 1 \\ x_1 & -2x_2 & -4x_3 & +5x_4 & = & 9 \end{array}$$

2. [9 points] Describe all matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ that commute with the matrix $A = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$.

   Note: There are infinitely many such matrices; one way to express your answer is to express a, b, c, d in terms of one or more free variables.

3. [9 points] Suppose that A is a 2×2 matrix that transforms the unit square in the plane in the manner shown below.





   (a) Determine A. There are more than one possible answer; you only need to give one.

   (b) Using the matrix A you found in part (a), determine the area of the parallelogram in the second picture.

4. [9 points] (a) Find the determinants of the following two matrices.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 7 \\ 1 & 2 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -1 & 5 \\ 1 & 0 & 9 \\ 2 & -1 & 14 \end{pmatrix}$$

   (b) One of the matrices A, B is invertible, while the other one is not. Determine which is which, and then compute the inverse of the one that is invertible.

5. [9 points] Consider the following four vectors.

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \vec{v}_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \qquad \vec{v}_3 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 8 \\ -3 \\ -1 \end{pmatrix}$$

   Express $\vec{b}$ as a linear combination of $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$.

6. [9 points] Suppose that A and B are both $n \times n$ matrices, and that the product AB is invertible. Prove that A is also invertible.