Study guide

- (§1.5) Understand how to convert a linear system of equations to a matrix equation ($A\vec{x} = \vec{b}$) and vice versa.
- (§1.4) Know the defintions of the identity matrix I_n (or I, when n is clear from context) and the inverse matrix A^{-1} (when it exists).
- (§1.4) Remember: only square matrices (same number of rows as number of columns) can possibly be invertible.
- (§1.4) Be able to quickly invert 2×2 matrices, e.g. using the formula mentioned in class.
- (§1.4) Be able to invert matrices of any size, and to tell when the inverse doesn't exist, by row-reducing a large matrix.
- (§1.5) Know how to use the inverse matrix (if it exists) to solve a matrix equation.

Textbook problems

- §1.4: 2, 6, 8, 11, 22
- §1.5: 14, 18, 30

Supplemental problems:

1. Suppose that A is an $n \times n$ matrix, $\vec{u}$ is a nonzero vector, and $A\vec{u} = \vec{0}$ (here, $\vec{0}$ denotes the $n \times 1$ column vector with all entries equal to 0). Prove that A is *not* invertible.

2. Suppose that A is an invertible matrix. Prove that the transpose A^t is also invertible, and that its inverse is given by $(A^{-1})^t$.

   Hint: You may wish to make use of some algebraic facts about transposition, summarized in Theorem 6 on page 36 of the textbook.

3. (a) Let A be an $n \times n$ matrix. Suppose that in every row of A, the entries of that row sum to 0. Prove that A is not invertible.

   Hint: Use the result of problem 1.

   (b) Suppose instead that *in every column* of A, the entries in that column sum to 0. Deduce from part (a) and problem 2 that A is not invertible.

   Note: To clarify the wording: part (a) concerns matrices like $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 1 & -4 & 3 \end{pmatrix}$ (note that in each of the three rows, the numbers sum to 0), while part (b) concerns matrices like $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & -4 \\ -3 & -1 & 3 \end{pmatrix}$.