**Textbook reading** for this week:

- Chapter 1.6 (basis and dimension)
- Suggested: begin reading 2.1 (linear transformations)

**Study items:**

- How do you determine whether a set of vectors spans all of $\mathbb{R}^n$?
- How do you determine if a given set is a basis for a vector space?
- How can you find a basis for a subspace of $\mathbb{R}^n$ given by a system of linear equations?
- Why is it possible to extend a given linearly independent set to a basis? How can you do it computationally?
- Be able to use the definition of basis to answer theoretical questions about vector spaces.
- Given a spanning set, how can you find a basis contained in it?
- Be able to use bases to solve theoretical problems about vector spaces.
- Know the definition of "coordinates with respect to a basis," and be able to use it. (This is not defined in the book until Chapter 2, but we discussed it in class while discussing bases)

**Problems:**

1. Let $p(x) = 5 + 2x - x^2$. This is an element of $P_2(\mathbb{R})$.
   (a) Let $B = \{1, x, x^2\}$. This is the *standard basis* of $P_2(\mathbb{R})$. Determine the coordinate vector $[p]_B$ for $p$ in this basis.
   (b) Let $B_1 = \{1, x-1, (x-1)^2\}$. This is also a basis of $P_2(\mathbb{R})$ (you don't need to prove this). Determine the coordinates $[p]_{B_1}$ in basis $B_1$.
2. Let $B_1 = \{1, x-1, (x-1)^2\}$ be the basis of $P_2(\mathbb{R})$ mentioned in the previous problem. Prove that for any $p \in P_2(\mathbb{R})$, the coordinates of $p$ in basis $B_1$ are given by the following formula.
$$[p]_{B_1} = \left(p(1), p'(1), \frac{1}{2}p''(1)\right).$$
   (This formula may remind you for Taylor expansions from calculus – this is not a coincidence.)
3. *(Damiano–Little 1.6.2(a,b,d))* (Basis and dimension for subspaces given by homogeneous equations)
4. *(Damiano–Little 1.6.14(a))* **First read the definition of the vector space $M_{m\times n}(\mathbb{R})$ on pages 20-21, between exercises 10 and 11.** (Basis and dimension for vector spaces of matrices)
5. *(Damiano–Little 1.6.15(a))* (Basis and dimension for a subspace of $M_{m\times n}(\mathbb{R})$)
6. *(Damiano–Little 1.6.3)* (a subspace must have smaller or equal dimension)

7. *(Damiano–Little 1.6.5(a,b))* (dimension of an intersection of subspaces)
8. *(Damiano–Little 1.6.7(b))* (extending from a linearly independent set to a basis)
9. *(Damiano–Little Chap 1 Supplementary (pp 59-61), 2(a,b))* (Basis from homogeneous system; relating it to an inhomogeneous system)
10. *(Damiano–Little Chap 1 Supplementary (pp 59-61), 3)* (basis for a subspace of $P_4(\mathbb{R})$ given by an equation)

**Extra practice** (not to hand in)

- *(Damiano–Little 1.6.8)*
- *(Damiano–Little 1.6.11)*
- *(Damiano–Little 1.6.13)*
- *(Damiano–Little 1.6.2(e,f))*
- *(Damiano–Little 1.6.7(a,c))*