Math 271, Linear Algebra, Fall 2022
Midterm 1 Practice Test 1

(This is a modified version of Harris Daniels's Midterm 1 practice test from Fall 2016)

Instructions:

- You may not use notes, books, calculators, cell phones or any other aids.
- You must show all your work to get full credit.
- You have 50 minutes to complete the exam.

Answer the following questions:

1. Let $V = \mathbb{R}^+$ be the vector space whose objects are the positive real numbers with addition and scalar multiplication operations defined by

$$x \oplus y = xy, \quad c \odot x = x^c.$$

   (a) Prove that 1 is the additive identity for the vector space V.

   (b) Prove one of the two distributive laws for the vector space V.

   (c) Prove $V = \text{span}(\{e\})$ where e is the base of the natural logarithm function $\ln(x)$.

2. Is the vector $(4, 0, 6, 9)$ in the span of the set $\{(2, 1, 0, 0), (0, 1, 0, 0), (0, 1, -2, -3)\}$? Justify your answer.

3. Put the following linear system into echelon form and use your answer to write down an expression for the solution set in terms of free variables

$$\begin{aligned} x_2 + 2x_3 - x_4 + x_5 &= 1 \\ x_1 \quad + 4x_3 \quad + x_5 &= 2 \\ x_1 - 2x_2 \quad\quad &= 0 \end{aligned}$$

4. Let V be a vector space and suppose $\{\mathbf{u}, \mathbf{v}\}$ is a basis for V. Prove that $\{2\mathbf{u} - \mathbf{v}, \mathbf{u} + \mathbf{v}\}$ is also a basis for V.

5. Define subspace of $P_2(\mathbb{R})$ by $W = \{f \in P_2(\mathbb{R}) \mid f'(2) = 0\}$. You may assume that W is indeed a subspace. Find a set that spans W. (Note: f' here refers to the derivative of f.)