Review Packet for Exam #1

Fall 2021, Math 121, Professor Danielle Benedetto

with some annotations by N. Pflueger, 2023.

Derivatives: Compute the derivative for each of the following functions. Do not worry about simplifying your answers:

1. $f(x) = \arcsin x \cdot \arctan x + \arctan(\sin(\ln x))$
2. $f(x) = \dfrac{\sinh(x^2 - 2)}{x + \sin^{-1} x}$
3. $f(x) = \sin\left(e^{\arcsin e^x}\right)$
4. $f(x) = \ln\left(1 - \arcsin\left(\dfrac{7}{x}\right)\right)$
5. Compute $f''(x)$ where $f(x) = \ln(1 - x)$.
6. Compute $f''(x)$ where $f(x) = \arctan(3x)$.
7. Compute $f''(x)$ where $f(x) = \arcsin(4x)$.

Proofs:

8. Prove that $\dfrac{d}{dx} \arcsin x = \dfrac{1}{\sqrt{1 - x^2}}$
9. Prove that $\dfrac{d}{dx} \arctan x = \dfrac{1}{1 + x^2}$
10. Prove that $\dfrac{d}{dx} \sinh x = \dfrac{1}{\sqrt{1 + x^2}}$
11. Prove that $\dfrac{d}{dx} \arcsin(5x) = \dfrac{5}{\sqrt{1 - 25x^2}}$
12. Prove that $\dfrac{d}{dx} \ln x = \dfrac{1}{x}$
13. Prove that $\dfrac{d}{dx} \arctan(3x) = \dfrac{3}{1 + 9x^2}$
14. Prove, using Integration, that $\displaystyle\int \frac{1}{9 + x^2}\, dx = \frac{1}{3} \arctan\left(\frac{x}{3}\right) + C$
15. Prove, using Integration, that $\displaystyle\int \frac{1}{\sqrt{16 - x^2}}\, dx = \arcsin\left(\frac{x}{4}\right) + C$

Limits: Compute each of the following limit.

16. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$

17. $\lim_{x\to 0}\frac{7xe^{x}-\arctan(7x)}{\sin x+\ln(1-x)}$

18. $\lim_{x\to 0}\frac{\cos(4x)-1-\arctan(4x)+4x}{\ln(1-x)+\arcsin x}$

19. $\lim_{x\to 0}\frac{1-e^{-4x}-\arctan(4x)}{x^{2}}$

20. $\lim_{x\to 0^{+}} x^{3}\ln x$

21. $\lim_{x\to 0^{+}} x\ln\left(\frac{1}{x}\right)$

22. $\lim_{x\to\infty} x^{\frac{1}{x^{2}}}$

23. $\lim_{x\to 0^{+}}(1-2x)^{\frac{1}{x}}$

24. $\lim_{x\to\infty}\left(x^{3}+1\right)^{\frac{1}{\ln x}}$

25. $\lim_{x\to\infty} x^{2}\sin\left(\frac{1}{x^{2}}\right)$

26. $\lim_{x\to 0^{+}}\sqrt{x}\ln x$

27. $\lim_{x\to\infty}\left(1-\frac{3}{x}\right)^{4x}$

28. $\lim_{x\to\infty}\left(1+\ln\left(1-\frac{6}{x^{2}}\right)\right)^{x^{2}}$

29. $\lim_{x\to\infty}\left(1-\arctan\left(\frac{5}{x^{4}}\right)\right)^{3x^{4}}$

30. CHALLENGE: $\lim_{x\to\infty}\left(\sqrt{1-\frac{5}{x^{2}}}-\sin\left(\frac{1}{x^{2}}\right)\right)^{5x^{2}}$

31. $\lim_{x\to\infty}\left(e^{\frac{1}{x^{8}}}-\frac{8}{x^{8}}\right)^{x^{8}}$

32. $\lim_{x\to\infty}\left(\frac{x}{x+3}\right)^{x}$

Integrals: Compute each of the following integrals.

33. $\displaystyle\int (e^x + x)^2 \; dx$

34. $\displaystyle\int x \sin^2 x \; dx$

35. $\displaystyle\int \frac{1}{\sqrt{25 - x^2}} \; dx$

36. $\displaystyle\int \frac{1}{x^2 + 25} \; dx$

37. $\displaystyle\int \frac{1}{x\sqrt{9 - \ln^2 x}} \; dx$

38. $\displaystyle\int x \arcsin x \; dx$

39. $\displaystyle\int \frac{1}{(4 - x^2)^{\frac{3}{2}}} \; dx$

40. $\displaystyle\int_1^e \ln x \; dx$

41. $\displaystyle\int \frac{\ln(2x^5)}{x^2} \; dx$

42. $\displaystyle\int \cos^5 x \; dx$

43. $\displaystyle\int \ln^2(x^{20}) \; dx$

44. $\displaystyle\int \sin^5 x \cos^2 x \; dx$

45. $\displaystyle\int \sin^2 x \cos^3 x \; dx$

46. $\displaystyle\int_{e^{\sqrt{3}}}^{e^3} \frac{1}{x(9 + (\ln x)^2)} \; dx$

47. $\displaystyle\int_1^e \frac{1}{x(1 + (\ln x)^2)^{\frac{3}{2}}} \; dx$

48. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1 + \sin^2 x)^{\frac{7}{2}}} \; dx$

49. $\displaystyle\int_1^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}} \; dx$

50. $\displaystyle\int \frac{1}{(x^2 + 4)^2} \; dx$

51. $\displaystyle\int \frac{1}{(x^2+4)^{\frac{7}{2}}}\ dx$

52. $\displaystyle\int x^4 \arcsin x\ dx$

53. $\displaystyle\int x \arctan x\ dx$

54. $\displaystyle\int_0^1 x \tan^{-1}(x^2)\ dx$

55. $\displaystyle\int \frac{x^2}{x^6+1}\ dx$

56. $\displaystyle\int_1^{e^2} x \ln\sqrt{x}\ dx$

57. $\displaystyle\int \frac{x^2}{(1-x^2)^{\frac{3}{2}}}\ dx$

58. $\displaystyle\int_1^e (\ln x)^2\ dx$

59. $\displaystyle\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} + \frac{1}{x^2+9}\ dx$

60. $\displaystyle\int \frac{x^2}{\sqrt{16-x^2}}\ dx$

61. $\displaystyle\int x^3\sqrt{9-x^2}\ dx$

62. $\displaystyle\int \frac{x^2}{x^2+3}\ dx$

63. $\displaystyle\int_{-3}^3 \sqrt{9-x^2}\ dx$

64. $\displaystyle\int_1^e \sqrt{x}\ln x\ dx$

65. $\displaystyle\int \frac{x+3}{\sqrt{4-x^2}}\ dx$

66. CHALLENGE: $\displaystyle\int (\arcsin x)^2\ dx$