Textbook problems:

- §7.4: 9, 12, 17, 40, 59, 60, 62
- §7.5: 4, 10, 44, 64, 70

Supplemental problems:

1. Evaluate the following integrals.

   (a) $\displaystyle\int \frac{1}{(x^2+25)^{3/2}}\,dx$

   (b) $\displaystyle\int \frac{x^3}{(x^2+1)^{5/2}}\,dx$

   (c) $\displaystyle\int_0^2 \frac{x^2}{(x^2+4)^2}\,dx$

   (d) $\displaystyle\int_0^1 \frac{x^2}{(x^2+1)^{7/2}}\,dx$

2. This sequence of exercises is a preview of a topic coming in a couple weeks. The aim is to give you an example, outside of mathematics itself, of some of the key ideas from our upcoming series unit.

   Read the excerpt on the second page of this pdf (the portion that is relevant for us ends halfway through the first paragraph of the second page). This is excerpted from *A Concise Guide to Macroeconomics* by David Moss, concerning the concept of an *income multiplier*. You do not need to understand all the details, but try to get the gist. Then answer the following questions.

   (a) The text asserts that "eventually, the increments would become too small to matter" (middle of the first page). In their example, how many increments must be included before the increments shrink to less than a dollar?

   (b) Suppose that the government initiates an additional \$128 in deficit spending, and assume that each household saves 75% of any additional income (rather than saving 20%, as in the book's example). What are the first five increments in GDP, and what is their sum?

   (c) In the situation of part (b), what is the income multiplier, and what does Keynes's formula suggest the *total* change in GDP would be, after all increments are taken into account?

Important notes:

- Regrade requests must be submitted via Gradescope within *one week of the due date* of the assignment.
- For full credit, you must show or explain your reasoning.
- You are encouraged to work in groups while solving the problems, but all submitted work must be your own work in your own words. Use of solution manuals or online solution databases is plagiarism, and will result in a 0 on the assignment in addition to being reported to Community Standards.

Submission instructions:
Before submitting your assignment scan it to a single pdf file and **view your pdf to make sure that it is clearly legible.** Then submit it as follows.

1. Go to `http://www.gradescope.com` and log in.
2. Select "Math 111" and the appropriate homework assignment, then select "submit pdf."
3. For each written question, select the pages of your submission where your solution appears.
4. Click submit.

For convenience, imagine that the effect worked only through consumption. Suppose further that households consumed 80 percent of every new dollar of income and saved the rest. In this case, if the government initiated an additional $100 in deficit spending, the first-round effect would be for GDP (national income) to rise by $100. Faced with an additional $100 of income, households would spend 80 percent of it, or $80, which would thus increase GDP by another $80. Now, with another $80 of income, households would spend 80 percent of it, or $64, which would again increase GDP by the same amount. So far, GDP would have increased by $244 (i.e., $100 + $80 + $64), based on the government's original $100 in deficit spending. But the process wouldn't end there. The virtuous circle would continue to go round and round and round, and GDP would continue to rise, with each increment of growth equal to 80 percent of the one before (i.e., $100 + $80 + $64 + $51.20 + $40.96 + $32.77 + $26.21 + ...). Eventually, the increments would become too small to matter. In the meantime, however, GDP would have grown by about $500. (See figure 3-1.) Keynes noted that the whole process can be boiled down to the following formula:

Change in GDP = (Change in deficit spending by government) × Income multiplier,

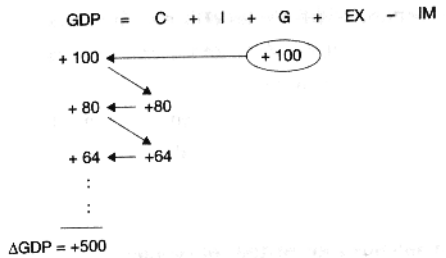
where the income multiplier = (1/proportion leakage from the income–expenditure cycle). In this case, since the proportion of leakage (that is, the amount of new income not spent) equals 20 percent (or 0.20), the income multiplier equals 1/0.20, or 5. Thus:

Change in GDP = $100 × 5 = $500.

FIGURE 3-1

Illustration of Keynesian income multiplier

Illustration is based on $100 increase in government deficit spending and leakage of 20%.



This analysis suggests that an increase in deficit spending will increase nominal GDP. In the example given, the increase in GDP is 5 times as large as the original increase in deficit spending, since leakage is only 20 percent and the income multiplier is therefore equal to 5. What is not clear, however, is whether the increase in *nominal* GDP will come mainly from an increase in the price level (P) or from an increase in the quantity of output (Q). Recall that nominal GDP depends on both of these variables, since nominal GDP = P × Q.

Keynes believed that in times of high unemployment, the increase would come mainly from the quantity of output (that is, mainly from an increase in *real* GDP). In a depression, with many productive resources lying idle, the first thing business managers would do in the face of increased demand would be to put idle resources back to work. They would rehire workers, turn equipment back on, and bring their factories back to life. As a result of all this, production (real GDP) would increase toward the economy's potential.