Evaluating limits: examples. // for practice in class; not to hand in.

Evaluate each limit, or explain why it doesn't exist.

① $\lim_{x \to 7} \frac{x-7}{|7-x|}$

② $\lim_{x \to 2} \frac{x^2-6x+8}{x-2}$

③ $\lim_{x \to 1} \frac{x^2+2x-3}{x^2-4x+3}$

④ $\lim_{x \to 7} \frac{x^2-10x+21}{|7-x|}$

⑤ $\lim_{t \to 0} \frac{1-\sqrt{1+t}}{t^2+t}$

Section 1.6 Calculating limits using the limits laws

Theorem. (*Limits Laws*) Assume that

$$\lim_{x\to a} f(x) \quad \text{and} \quad \lim_{x\to a} g(x)$$

exist. Then,

1. Sum Law $\lim\limits_{x\to a}\big[f(x)+g(x)\big] = \lim\limits_{x\to a} f(x) + \lim\limits_{x\to a} g(x)$
2. Difference Law $\lim\limits_{x\to a}\big[f(x)-g(x)\big] = \lim\limits_{x\to a} f(x) - \lim\limits_{x\to a} g(x)$
3. Constant Multiple Law $\lim\limits_{x\to a}\big[cf(x)\big] = c\lim\limits_{x\to a} f(x)$, where, c is a constant.
4. Product Law $\lim\limits_{x\to a}\big[f(x)g(x)\big] = \lim\limits_{x\to a} f(x)\cdot\lim\limits_{x\to a} g(x)$
5. Quotient Law $\lim\limits_{x\to a}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$ provided that $\lim\limits_{x\to a} g(x) \neq 0$

Some of the following laws can be derived from other laws (e.g, laws 6, 9) or by illustration (e.g., laws 7, 8).

Theorem. (*Further Limits Laws*) Assume that $\lim\limits_{x\to a} f(x)$ exists. Then,

6. $\lim\limits_{x\to a}\big[f(x)\big]^n = \big[\lim\limits_{x\to a} f(x)\big]^n$
7. $\lim\limits_{x\to a} c = c$
8. $\lim\limits_{x\to a} x = a$
9. $\lim\limits_{x\to a} x^n = a^n$
10. $\lim\limits_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$, where n is a positive integer, and if n is even, then $a > 0$
11. $\lim\limits_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x\to a} f(x)}$, where n is a positive integer