① $f(x) = (x+1)^2 (x-2)^3$

a) $f'(x) = 2(x+1)(x-2)^3 + (x+1)^2 \cdot 3(x-2)^2$

$$= (x+1)(x-2)^2 \cdot \left[2(x-2) + 3(x+1)\right]$$

$$= (x+1)(x-2)^2 \cdot (5x-1)$$

b) $\boxed{x = -1,\ \frac{1}{5},\ \&\ 2}$ (no pts. where $f'(x)$ is undefined)

c)

	$x<-1$	$-1<x<1/5$	$1/5<x<2$	$x>2$
$(x+1)$	−	+	+	+
$(x-2)^2$	+	+	+	+
$(5x-1)$	−	−	+	+
f'	+	−	+	+
f	↗	↘	↗	↗

(squares aren't negative)

max at -1, min at $1/5$ (1st deriv. test)

Local max @ $x = -1$ ($y = 0$)
Local min @ $x = 1/5$ ($y = (\frac{6}{5})^2(-\frac{9}{5})^3 \approx -8.40$)

increasing on $(-\infty, -1)$ & $(1/5, \infty)$
decreasing on $(-1, 1/5)$.

② $g(x) = \frac{1}{1-x^2}$

a) $g'(x) = -\frac{1}{(1-x^2)^2} \cdot (-2x) = \frac{2x}{(1-x^2)^2}$

b) $\boxed{x=0}$ (where $g'(x)=0$). Note that $g'(x)$ is undefined at $\underline{x=\pm 1}$, but these are not technically crit. numbers since they aren't in the domain of g itself. But they are still spots where f may change from inc. to dec. (& vice versa).

c)

	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
$2x$	−	−	+	+
$\frac{1}{(1-x^2)^2}$	+	+	+	+
f'	−	−	+	+
f	↘	↘	↗	↗

min (1st deriv test) at $x = 0$

inc. on $(0,1)$ & $(1,\infty)$
dec on $(-\infty,-1)$ & $(-1,0)$
Local min. @ $x=0$
$(y=1)$

③ $h(x) = x^{2/3}(2-x)$

a) $h'(x) = \frac{2}{3}x^{-1/3}(2-x) + x^{2/3}\cdot(-1)$

$= \frac{2}{3}\cdot 2x^{-1/3} - \frac{2}{3}x^{2/3} - x^{2/3} = \frac{4}{3}x^{-1/3} - \frac{5}{3}x^{2/3}$

$= \frac{1}{3}x^{-1/3}\cdot\left[4 - 5x\right]$

crit. pts: $\boxed{x=0 \ \& \ x = 4/5}$ ($x=0$: where $h'(x)$ is undef. due to div. by 0); ($x=4/5$: where $4-5x=0$)

+/− analysis:

	$(-\infty,0)$	$(0,4/5)$	$(4/5,\infty)$
$\frac{1}{3}x^{-1/3}$	−	+	+
$4-5x$	+	+	−
h'	−	+	−
h	↘	↗	↘

min at $x=0$, max at $x=4/5$ (1st deriv. test)

inc. on $(0,4/5)$
dec. on $(-\infty,0)$ & $(4/5,\infty)$
Local min @ $x=0$
Local max @ $x=4/5$

④

a) $f'(x)$ never undef. (as far as the graph shows).

$f'(x) = 0$ at $\boxed{x = -2,\ 0,\ \&\ 2}$

(where $y = f'(x)$ crosses the axis).

b) $x = -2$ is a local min (f' changes from $-$ to $+$, so f changes from ↘ to ↗).

$x = 0$ is a local max (f' changes $+$ to $-$)

$x = 2$ is a local min (f' changes from $-$ to $+$).

c) $f''(-2) > 0$ since $f'(x)$ is increasing at $x = -2$.

So $f(x)$ is <u>concave up</u> at $x = -2$.

⑤ $f(x) = x^3 + 3x^2 - 1$

a) $f'(x) = 3x^2 + 6x$

$= 3x(x+2)$

b) $\boxed{x = 0\ \&\ -2}$ where $f'(x) = 0$ (undefined nowhere)

c)

	$x<-2$	$-2<x<0$	$x>0$
$3x$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$
f'	$+$	$-$	$+$
f	↗	↘	↗

max at -2, min at 0 (1st deriv. test)

inc. on $(-\infty, -2)$ & $(0, \infty)$

dec. on $(-2, 0)$

Local max $(-2, 3)$ @ $x = -2$, $y = f(-2) = -8 + 12 - 1 = \underline{3}$

Local min $(0, -1)$ @ $x = 0$, $y = -1$

d) $f''(x) = 6x + 6 = 6(x+1)$

e) $f''(x) = 0$ @ $x = -1$, & f'' is never undefined.

	$x<-1$	$x>-1$
$6(x+1)$	$-$	$+$
f is	∩ con. down	∪ con. up

conc. up on $(-1, \infty)$

conc. down on $(-\infty, -1)$

inflection @ $x = -1$, $y = f(-1) = -1 + 3 - 1 = 1$

ie. $(-1, 1)$

(b) For each critical number you found, determine whether it is a local max, local min, or neither.

(c) Is $f''(-2)$ positive or negative? From this, what can you say about the concavity of the original function $f(x)$ at $x = -2$?

5. Let $f(x) = x^3 + 3x^2 - 1$.

(a) Compute $f'(x)$ and factor it.

(b) Determine the critical numbers of $f(x)$.

(c) Find the intervals on which $f(x)$ is increasing and decreasing, and list any local min(s) and max(s). Give both the x and y-coordinates.

(d) Compute $f''(x)$.

(e) Find the intervals on which $f(x)$ is concave up and concave down, and list any inflection point(s). Give both x and y-coordinates for these.

(f) Plot the local min(s) and max(s) you found in in parts (5c) and (5e) on the axes below (or on your own sheet of paper). Use these to give a rough sketch of the curve. Make sure it is increasing/decreasing on the right intervals, and concave up/down in the right places.