

WORKSHEET 5 SOLUTIONS

$$1) f'(x) = \frac{(x+1)'(x-1) - (x+1)(x-1)'}{(x-1)^2} = \frac{(x-1) - (x+1)}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2}$$

$$= \frac{-2}{(x-1)^2}$$

$$2) y'(x) = \frac{-2}{(x-1)^2}; \text{ NOTE: } y(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2 \text{ and } y'(3) = \frac{-2}{(3-1)^2} = \frac{-2}{2^2}$$

So, we have $(3, 2)$ and $y'(3) = -1/2$,
 which gives us $y - 2 = -1/2(x - 3) \Rightarrow y - 2 = -1/2x + 3/2$

$$\boxed{y = -1/2x + 7/2}$$

$$3) f'(x) = (x^2+7)' \sqrt{4x+1} + (x^2+7)(\sqrt{4x+1})' \leftarrow \text{this was given!}$$

$$= 2x \sqrt{4x+1} + (x^2+7) \cdot \frac{2}{\sqrt{4x+1}}$$

$$= \frac{\sqrt{4x+1} (2x) \sqrt{4x+1}}{\sqrt{4x+1}} + \frac{2(x^2+7)}{\sqrt{4x+1}} = \frac{2x(4x+1) + 2x^2 + 14}{\sqrt{4x+1}}$$

$$= \frac{8x^2 + 2x + 2x^2 + 14}{\sqrt{4x+1}} = \frac{10x^2 + 2x + 14}{\sqrt{4x+1}}$$

$$4) \frac{d}{dx} \left(\frac{2x+1}{3x+1} \right) = \frac{(2x+1)'(3x+1) - (2x+1)(3x+1)'}{(3x+1)^2} = \frac{2(3x+1) - 3(2x+1)}{(3x+1)^2}$$

$$= \frac{6x+2-6x-3}{(3x+1)^2} = \frac{-1}{(3x+1)^2}$$

$$5) f(x) = \frac{3x^4}{x\sqrt{x}} + \frac{2x^2}{x\sqrt{x}} - \frac{7x}{x\sqrt{x}} + \frac{1}{x\sqrt{x}} = \frac{3x^4}{x^{3/2}} + \frac{2x^2}{x^{3/2}} - \frac{7x}{x^{3/2}} + \frac{1}{x^{3/2}}$$

$$= 3x^{5/2} + 2x^{1/2} - 7x^{-1/2} + x^{-3/2}$$

$$f'(x) = \frac{5}{2}(3)x^{3/2} + 2\left(\frac{1}{2}\right)x^{-1/2} - 7\left(-\frac{1}{2}\right)x^{-3/2} + \left(-\frac{3}{2}\right)x^{-5/2}$$

$$= \frac{15}{2}x^{3/2} + \frac{1}{\sqrt{x}} + \frac{7}{2x\sqrt{x}} - \frac{3}{2x^2\sqrt{x}}$$

b.) After 2 hours, $\frac{1}{4}(2)^2 = 1$, gives us 1 inch of rainfall. So, the average = $1/2$ (hours)
 $\Rightarrow \boxed{1/2}$ inches per hour.

b) How fast the rain is falling (velocity) indicates that we are looking at a derivative/slope.

$$\left(\frac{1}{4}t^2\right)' = \frac{1}{4}(2)t' = \frac{1}{2}t.$$

So after 2 hours, velocity is $\frac{1}{2}(2) = \boxed{1}$ inch/hour.

$$7) \left. \begin{array}{l} f(2) = 5 \quad g(2) = 7 \\ f'(2) = -1 \quad g'(2) = 2 \end{array} \right\} (f \cdot g)'(2) = f'(2)g(2) + f(2)g'(2) \\ = (-1)(7) + (5)(2) = -7 + 10 = \boxed{3}$$

$$(f/g)'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{(g(2))^2}$$

$$= \frac{(-1)(7) - (5)(2)}{(7)^2} = \boxed{\frac{-17}{49}}$$

8.a) It is not possible to cancel $(2x^2-3)$ in this fraction because we cannot factor out a $(2x^2-3)$ from the numerator, since it is not in both terms.

$$b) f'(x) = \frac{8x^5 - 12x^3 - 4x^5}{(2x^2-3)^2} = \frac{4x^5 - 12x^3}{(2x^2-3)^2} = \boxed{\frac{4x^3(x^2-3)}{(2x^2-3)^2}}$$

$$c) (\text{From hint}) 0 = 4x^3(x^2-3)$$

This is true when $4x^3 = 0$ OR $x^2 - 3 = 0$

$\Rightarrow f'(x) = 0$ when $x = 0$ OR $x = \pm\sqrt{3}$.

d) For this, plug the x -values we found in (c) into the original equation:

$$f(0) = \frac{(0)^4}{2(0)^2-3} = 0, \quad f(\sqrt{3}) = \frac{(\sqrt{3})^4}{2(\sqrt{3})^2-3} = \frac{9}{6-3} = 3$$

$$f(-\sqrt{3}) = \frac{(-\sqrt{3})^4}{2(-\sqrt{3})^2-3} = \frac{9}{3} = 3 \quad \text{so, } \boxed{(0,0), (\sqrt{3},3), (-\sqrt{3},3)}$$

9) Remember that a horizontal line means that the slope is 0.
So, again like #8, we want $f'(x) = 0$.

$$y'(x) = \frac{(x)(x^2+1) - (x^2+1)(x)'}{(x^2+1)^2} = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

So, we want to find $(1-x^2) = 0$

$$\Rightarrow (1-x)(1+x) = 0$$

We can have $(1-x) = 0$ or $(1+x) = 0$, so $x = \underline{\underline{1, -1}}$ give us 0.

$$y(1) = \frac{1}{1^2+1} = \frac{1}{2}$$

$$y(-1) = \frac{-1}{(-1)^2+1} = \frac{-1}{2}$$

So the points are $\boxed{(1, \frac{1}{2})}$ and $\boxed{(-1, -\frac{1}{2})}$.