

# WORKSHEET 4 - SOLUTIONS:

Recall:  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$\begin{aligned}
 1. \text{ (a)} \quad f'(1) &= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{5 - 6(1+h) + 4(1+h)^2 - (5 - 6 + 4)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{5} - \cancel{6} - 6h + 4(1 + 2h + h^2) - \cancel{5} + \cancel{6} - 4}{h} = \lim_{h \rightarrow 0} \frac{-6h + 4 + 8h + 4h^2 - 4}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{h}(2 + 4h)}{\cancel{h}} = \lim_{h \rightarrow 0} 2 + 4h = \boxed{2}
 \end{aligned}$$

(b) By part (a), slope is 2.  
 $x = \underline{1} \Rightarrow f(1) = 5 - 6 + 4 = \underline{3}$  } 
$$\begin{aligned}
 y - 3 &= 2(x - 1) \\
 y &= 2x - 2 + 3 \\
 \boxed{y} &= \boxed{2x + 1}
 \end{aligned}$$

$$\begin{aligned}
 2. \text{ (a)} \quad f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h)(x^2 + 2xh + h^2) - x^3}{h} = \lim_{h \rightarrow 0} \frac{\cancel{x^3} + 2x^2h + xh^2 + \cancel{x^2}h + 2xh^2 + \cancel{h^3} - x^3}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h \rightarrow 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 = \boxed{3x^2}
 \end{aligned}$$

$$\text{(b)} \quad f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \lim_{h \rightarrow 0} \frac{\cancel{x+h} - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

$$\text{(c)} \quad f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h} = \lim_{h \rightarrow 0} \frac{\cancel{-h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \boxed{\frac{-1}{x^2}}$$

$$\begin{aligned}
 (d) f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h} = \lim_{h \rightarrow 0} \frac{(x-1)(x+h+1) - (x+h-1)(x+1)}{(x+h-1)(x-1)h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{x^2} + \cancel{hx} + \cancel{x} - \cancel{x} - \cancel{h} - \cancel{1} - (\cancel{x^2} + \cancel{hx} + \cancel{x} + \cancel{h} - \cancel{x} - \cancel{1})}{(x+h-1)(x-1)h} \cdot \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-2h}{(x+h-1)(x-1)h} = \lim_{h \rightarrow 0} \frac{-2}{(x+h-1)(x-1)} = \boxed{\frac{-2}{(x-1)^2}}
 \end{aligned}$$

$$\begin{aligned}
 (e) f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} = \lim_{h \rightarrow 0} \frac{x - (x+h)}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \\
 &= \lim_{h \rightarrow 0} \frac{-h}{h(\sqrt{x}\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})} = \lim_{h \rightarrow 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} = \frac{-1}{(\sqrt{x})^2(\sqrt{x} + \sqrt{x})} = \boxed{\frac{-1}{2x\sqrt{x}}}
 \end{aligned}$$

[Note!  $2x\sqrt{x} = 2\sqrt{x^3}$  because  $2\sqrt{x}x^2$  would cause  $x^2$  to come out as  $\sqrt{x}$ ]

$$\text{So, } \frac{-1}{2x\sqrt{x}} = \frac{-1}{2\sqrt{x^3}}$$

$$3(a) 9x^2y + 2xy^3 = \boxed{xy(9x + 2y^2)}$$

$$(b) 2(x+1)^2y^3 - 8(x+1)y^5 = \boxed{2(x+1)y^3[(x+1) - 4y^2]}$$

$$\begin{aligned}
 (c) 3(x+1)^2(1-2x)^4 + (x+1)^34(1-2x)^3(-2) \\
 = (x+1)^2(1-2x)^3 [3(1-2x) + (x+1)4(-2)] = (x+1)^2(1-2x)^3 [3-6x-8x-8] \\
 = \boxed{(x+1)^2(1-2x)^3(-5-14x)}
 \end{aligned}$$

4(a) Remember  $f'(x)$  is the slope of the tangent line at  $x$ .

So  $f'(x) = 0$  when the tangent line slope is FLAT.

$f'(x) > 0$  "

$f'(x) < 0$  "

" is positive

" is negative.

Therefore,  $f'(x) = 0$  when  $x = 2, 6$ ;

$f'(x) > 0$  for  $x$  in  $[0, 2)$  and  $(6, \infty)$ .  $f'(x) < 0$  for  $x$  in  $(2, 6)$ .

(b) See graph for sketches.

Approximations will vary!!

$$f'(0) = 2$$

$$f'(2) = 0$$

$$f'(4) = -2$$

$$f'(6) = 0$$

$$f'(8) = 0.90$$

$$f'(10) = 0.95$$

}  $f'(x)$  for  $x > 6$ , should be increasing SLIGHTLY as  $x$  gets larger.

(c) See graph for rough sketch!