

# WORKSHEET 3: SOLUTIONS

$$1. \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \frac{0}{0} \quad \text{So } \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x \rightarrow 2} x+2 = \boxed{4}$$

$$\lim_{x \rightarrow 1} \frac{x^2 - 4}{x - 2} = \frac{1 - 4}{1 - 2} = \frac{-3}{-1} = \boxed{3}$$

The second one was easier to find b/c we didn't need to factor.

$$2. \lim_{x \rightarrow 2} \frac{x^2 + 6x + 8}{x - 2} = \frac{4 + 12 + 8}{0^-} = \frac{24}{0^-} \rightarrow \boxed{-\infty}$$

$$3. \lim_{x \rightarrow 2} \frac{x^2 + 5x - 14}{x^2 - 4x + 12} = \frac{4 + 10 - 14}{4 - 8 + 12} = \frac{0}{8} = \boxed{0}$$

$$4. \lim_{x \rightarrow 2} \frac{x^2 + 5x - 14}{x^2 - 8x + 12} = \frac{0}{0} = \lim_{x \rightarrow 2} \frac{(x+7)(x-2)}{(x-6)(x-2)} = \lim_{x \rightarrow 2} \frac{x+7}{x-6} = \frac{2+7}{2-6} = \frac{9}{-4} = \boxed{-\frac{9}{4}}$$

$$5. \lim_{x \rightarrow 0} \frac{x+1}{x(x+2)} \rightarrow \left. \begin{array}{l} \lim_{x \rightarrow 0^+} \frac{x+1}{x(x+2)} = \frac{1}{0^+} = +\infty \\ \lim_{x \rightarrow 0^-} \frac{x+1}{x(x+2)} = \frac{1}{0^-} = -\infty \end{array} \right\} \text{Since these are different, } \lim_{x \rightarrow 0} \frac{x+1}{x(x+2)} \text{ DNE}$$

$$6. \lim_{x \rightarrow 3^+} \frac{x^2 + 4x + 3}{x^2 - 2x - 15} = \frac{9 + 12 + 3}{9 + 6 - 15} = \frac{0}{0} \quad \text{So, } \lim_{x \rightarrow 3^+} \frac{x^2 + 4x + 3}{x^2 - 2x - 15} = \lim_{x \rightarrow 3^+} \frac{(x+3)(x+1)}{(x-5)(x+3)} \\ = \lim_{x \rightarrow 3^+} \frac{x+1}{x-5} = \frac{-3+1}{-3-5} = \frac{-2}{-8} = \boxed{\frac{1}{4}}$$

$$7. \lim_{x \rightarrow 3} \frac{x^2 + 4x + 3}{x^2 + 6x + 9} = \frac{9 + 12 + 3}{9 + 18 + 9} = \frac{0}{0} \quad \text{So } \lim_{x \rightarrow 3} \frac{x^2 + 4x + 3}{x^2 + 6x + 9} = \lim_{x \rightarrow 3} \frac{(x+3)(x+1)}{(x+3)(x+3)} \\ = \lim_{x \rightarrow 3} \frac{x+1}{x+3} = \frac{-2}{0} \Rightarrow \left. \begin{array}{l} \lim_{x \rightarrow 3^-} \frac{x+1}{x+3} = \frac{-2}{0^-} = +\infty \\ \lim_{x \rightarrow 3^+} \frac{x+1}{x+3} = \frac{-2}{0^+} = -\infty \end{array} \right\} \text{Since these are diff. } \lim_{x \rightarrow 3} \frac{x^2 + 4x + 3}{x^2 + 6x + 9} \text{ [DNE]}$$

$$8. \lim_{t \rightarrow 1} \frac{t-1}{g(t^2)-3}, \quad g(t) = 2t+1 \rightarrow \lim_{t \rightarrow 1} \frac{t-1}{g(t^2)-3} = \lim_{t \rightarrow 1} \frac{t-1}{(2t^2+1)-3} = \lim_{t \rightarrow 1} \frac{t-1}{2t^2-2} \\ = \lim_{t \rightarrow 1} \frac{t-1}{2(t^2-1)} = \lim_{t \rightarrow 1} \frac{t-1}{2(t+1)(t-1)} = \lim_{t \rightarrow 1} \frac{1}{2(t+1)} = \boxed{\frac{1}{4}}$$

$$= \frac{\frac{1}{9} - \frac{1}{9}}{0} = \frac{0}{0}$$

$$\begin{aligned} 9. \lim_{x \rightarrow -5} \frac{1}{4-x} - \frac{1}{9} &= \lim_{x \rightarrow -5} \frac{9}{9(4-x)} - \frac{4-x}{9(4-x)} = \lim_{x \rightarrow -5} \frac{9-4+x}{9(4-x)} \\ &= \lim_{x \rightarrow -5} \frac{5+x}{9(4-x)} \cdot \frac{1}{x+5} = \lim_{x \rightarrow -5} \frac{1}{9(4-x)} = \frac{1}{9(9)} = \boxed{\frac{1}{81}} \end{aligned}$$

$$10. \lim_{x \rightarrow -3} \frac{x^2 - 4x - 21}{\sqrt{1-x} - 2} = \frac{9 + 12 - 21}{\sqrt{1-3} - 2} = \frac{0}{0}$$

$$\hookrightarrow \lim_{x \rightarrow -3} \frac{x^2 - 4x - 21}{\sqrt{1-x} - 2} \cdot \frac{\sqrt{1-x} + 2}{\sqrt{1-x} + 2} = \lim_{x \rightarrow -3} \frac{(x^2 - 4x - 21)(\sqrt{1-x} + 2)}{(\sqrt{1-x})^2 - 4}$$

$$= \lim_{x \rightarrow -3} \frac{(x^2 - 4x - 21)(\sqrt{1-x} + 2)}{-3 - x} \stackrel{0/0}{=} \lim_{x \rightarrow -3} \frac{(x-7)(x+3)(\sqrt{1-x} + 2)}{-(x+3)}$$

$$= \lim_{x \rightarrow -3} \frac{(x-7)(\sqrt{1-x} + 2)}{-1} = \frac{(-3-7)(\sqrt{1-3} + 2)}{-1} = -(-10)(4) = \boxed{40}$$

$$11. f(x) = \frac{1}{x} \Rightarrow \lim_{t \rightarrow 2} \frac{f(t-1) - 2f(t)}{t^2 - 4} = \lim_{t \rightarrow 2} \frac{\frac{1}{t-1} - 2\left(\frac{1}{t}\right)}{t^2 - 4}$$

$$= \lim_{t \rightarrow 2} \frac{t - 2(t-1)}{t(t-1)(t+2)} = \lim_{t \rightarrow 2} \frac{t - 2t + 2}{t(t-1)(t+2)} = \lim_{t \rightarrow 2} \frac{-t + 2}{t(t-1)(t+2)}$$

$$= \lim_{t \rightarrow 2} \frac{-(t-2)}{t(t-1)(t+2)} = \lim_{t \rightarrow 2} \frac{-1}{t(t-1)(t+2)} = \frac{-1}{2(1)(4)} = \boxed{\frac{-1}{8}}$$

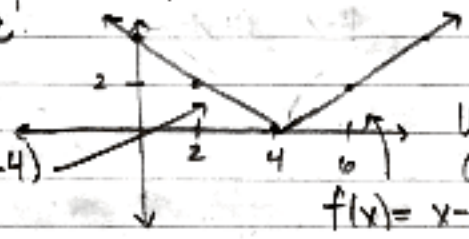
$$12. g(x) = \sqrt{x} \Rightarrow \lim_{s \rightarrow 1} \frac{g(s^2+8) - 3}{s-1} = \lim_{s \rightarrow 1} \frac{\sqrt{s^2+8} - 3}{s-1} = \frac{\sqrt{9} - 3}{0} = \frac{0}{0}$$

$$= \lim_{s \rightarrow 1} \frac{\sqrt{s^2+8} - 3}{s-1} \cdot \frac{\sqrt{s^2+8} + 3}{\sqrt{s^2+8} + 3} = \lim_{s \rightarrow 1} \frac{s^2+8-9}{(s-1)(\sqrt{s^2+8} + 3)} = \lim_{s \rightarrow 1} \frac{(s-1)(s+1)}{(s-1)(\sqrt{s^2+8} + 3)}$$

$$= \lim_{s \rightarrow 1} \frac{s+1}{\sqrt{s^2+8} + 3} = \frac{2}{\sqrt{9} + 3} = \frac{2}{6} = \boxed{\frac{1}{3}}$$

$$13. \lim_{x \rightarrow 4^-} \frac{|x-4|}{x-4}$$

Note: For this problem, think of what  $|x-4|$  looks like.



So,  $\lim_{x \rightarrow 4^-} \frac{|x-4|}{x-4} = \lim_{x \rightarrow 4^-} \frac{-(x-4)}{x-4} = \boxed{-1}$

$$14. \lim_{x \rightarrow 4^+} \frac{|x-4|}{x-4} = \lim_{x \rightarrow 4^+} \frac{x-4}{x-4} = \boxed{1}$$

$$15. \lim_{x \rightarrow 3} \frac{x}{x-2} - \frac{x+6}{x} = \frac{3}{1} - \frac{9}{3} = \frac{0}{0}$$

$$\lim_{x \rightarrow 3} \frac{x(x) - (x+6)(x-2)}{x(x-2)} = \lim_{x \rightarrow 3} \frac{x^2 - (x^2 + 4x - 12)}{x(x-2)} = \lim_{x \rightarrow 3} \frac{-4x + 12}{x(x-2)}$$

$$= \lim_{x \rightarrow 3} \frac{-4(\cancel{x-3})}{x(x-2)} \cdot \frac{1}{\cancel{x-3}} = \lim_{x \rightarrow 3} \frac{-4}{x(x-2)} = \frac{-4}{3(3-2)} = \boxed{\frac{-4}{3}}$$

$$16. \lim_{x \rightarrow 3} \frac{-1}{(x-3)^2} = \frac{-1}{0} \Rightarrow \text{So, } \lim_{x \rightarrow 3^-} \frac{-1}{(x-3)^2} = \frac{-1}{(0^-)^2} = -\infty$$

$$\lim_{x \rightarrow 3^+} \frac{-1}{(x-3)^2} = \frac{-1}{(0^+)^2} = -\infty$$

Since they're the same,  
 $\lim_{x \rightarrow 3} \frac{-1}{(x-3)^2} = \boxed{-\infty}$