

WORKSHEET 2 SOLUTIONS:

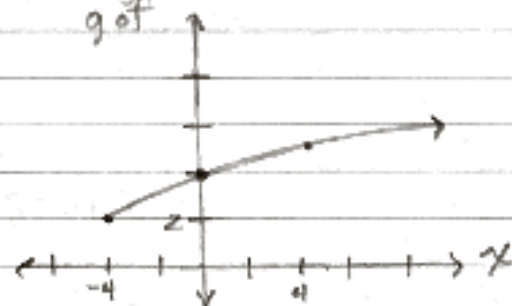
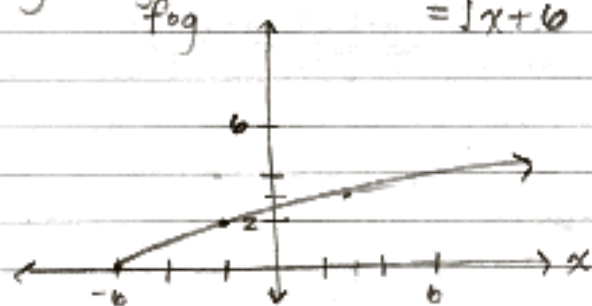
$$1. g(x) = \frac{x}{x-1} - \frac{x+2}{x} \Rightarrow \frac{x(x) - (x-1)(x+2)}{x(x-1)} \Rightarrow \frac{x^2}{x(x-1)} - \frac{x^2+x-2}{x(x-1)}$$

$$= \frac{x^2 - x^2 - x + 2}{x(x-1)} = \frac{-x+2}{x(x-1)} = \frac{-x+2}{x(x-1)} \cdot \frac{1}{x-2} = \frac{-(x-2)}{x(x-1)(x-2)} = \frac{-1}{x^2-x}$$

2. $f \circ g(x) = f(g(x))$

(a) Note that the domain of a function is everything the function can take in. So, the domain of $f(g(x))$ will be defined by $g(x)$ - that's what goes in. Since $g(x)$ is a value, specifically any value given by plugging in x , the domain of $f(g(x))$ is the range of $g(x)$.

$$(b) f \circ g = f(g(x)) = f(x+2) = \sqrt{(x+2)+4} = \sqrt{x+6} \quad | \quad g \circ f = g(f(x)) = g(\sqrt{x+4}) = \sqrt{x+4} + 2$$



$f \circ g$ does NOT equal $g \circ f$. Consider $x=0$

$$\left. \begin{array}{l} f(g(0)) = \sqrt{6} \\ g(f(0)) = 4 \end{array} \right\} \text{These are not equal!}$$

3. $f(f(2)) = \frac{f(2)+1}{f(2)-1}$ → You can break this up! $f(2) = \frac{2+1}{2-1} = \frac{3}{1} = 3$

Remember to plug all of this in first!

$$\text{So } \frac{f(2)+1}{f(2)-1} = \frac{3+1}{3-1} = \frac{4}{2} = 2$$

$$f(f(x)) = \frac{f(x)+1}{f(x)-1} = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} = \frac{\frac{x+1}{x-1} + \frac{x-1}{x-1}}{\frac{x+1}{x-1} - \frac{x-1}{x-1}} = \frac{\frac{x+1+x-1}{x-1}}{\frac{x+1-x+1}{x-1}} = \frac{2x}{2} = \frac{2x}{2}$$

$$= \frac{2x}{2} \cdot \frac{x-1}{2} = \frac{2x}{2} = \boxed{x}$$

$$4. f(x) = \frac{1}{x+1} \Rightarrow \frac{f(x+h) - f(x)}{h} = \frac{1}{(x+h)+1} - \frac{1}{x+1} = \frac{x+1}{(x+1)(x+h+1)} - \frac{x+h+1}{(x+1)(x+h+1)}$$

$$= \frac{x+1 - x-h-1}{(x+1)(x+h+1)} = \frac{-h}{(x+1)(x+h+1)} = \frac{-h}{(x+1)(x+h+1)} \cdot \frac{1}{h} = \boxed{\frac{-1}{(x+1)(x+h+1)}}$$

$$5. f(x) = \frac{x-7}{x+3} \Rightarrow \frac{f(x+h) - f(x)}{h} = \frac{x+h-7}{x+h+3} - \frac{x-7}{x+3} = \frac{(x+3)(x+h-7) - (x-7)(x+h+3)}{(x+3)(x+h+3)}$$

$$= \frac{x^2 + xh - 7x + 3x + 3h - 21 - (x^2 + xh + 3x - 7x - 7h - 21)}{(x+3)(x+h+3)} \quad \text{+ Remember to distribute}$$

$$= \frac{3h + 7h}{(x+3)(x+h+3)} = \frac{10h}{(x+3)(x+h+3)} \cdot \frac{1}{h} = \boxed{\frac{10}{(x+3)(x+h+3)}}$$

$$6(a) \frac{x^2 + 6x + 8}{x^2 - 4} = \frac{(x+2)(x+4)}{(x+2)(x-2)} = \boxed{\frac{x+4}{x-2}}$$

$$(b) \frac{x^2 + 6x + 8}{x^2 - 5x - 14} = \frac{(x+2)(x+4)}{(x-7)(x+2)} = \boxed{\frac{x+4}{x-7}}$$

$$(c) \frac{x^2 - 6x + 8}{y^2 - x - 2} = \frac{(x-4)(x-2)}{(x-2)(y+1)} = \boxed{\frac{y-4}{y+1}}$$

$$(d) \frac{1}{t|1+t} - \frac{1}{t} = \frac{1}{t|1+t} - \frac{|1+t|}{t|1+t} = \boxed{\frac{1 - |1+t|}{t|1+t}}$$

$$(e) \frac{t-1}{g(t^2)-3} = \frac{t-1}{2t^2+1-3} = \frac{t-1}{2t^2-2} = \frac{t-1}{2(t^2-1)} = \frac{t-1}{2(t-1)(t+1)} = \boxed{\frac{1}{2(t+1)}}$$

$$(f) \frac{x^2 - 13x + 42}{x^2 - 4x + 12} = \frac{(x-7)(x-6)}{x^2 - 4x + 12} \quad \left. \begin{array}{l} \text{can't simplify} \\ \text{any more?} \end{array} \right\} \text{Check: } \frac{4 \pm \sqrt{16 - 4(12)}}{2}$$

$$= \frac{4 \pm \sqrt{-32}}{2} \leftarrow \text{this neg!}$$

This tells us that there are no real solutions to this, so it can't be simplified more!

(g) $\frac{1}{x} - \frac{1}{|x|} \Rightarrow$ Case 1: $|x|$, x is pos.
 $\hookrightarrow \frac{1}{x} - \frac{1}{x} = \frac{0}{x} = \boxed{0}$

Case 2: $|x|$, x is neg.
 $\hookrightarrow \frac{1}{x} - \left(\frac{1}{-x}\right) = \frac{1}{x} + \frac{1}{x} = \boxed{\frac{2}{x}}$

h) $\frac{|x+4|}{x+4} \Rightarrow$ Case 1: $|x+4|$, $x+4$ is pos.
 $\hookrightarrow \frac{x+4}{x+4} = \boxed{1}$

Case 2: $|x+4|$, $x+4$ is neg.
 $\hookrightarrow \frac{-(x+4)}{x+4} = \boxed{-1}$

(i) $f(x) = \frac{1}{x}$, $\frac{f(t-1) - 2f(t)}{t^2 - 4} = \frac{\frac{1}{t-1} - 2\left(\frac{1}{t}\right)}{t^2 - 4} = \frac{\frac{t - 2(t-1)}{(t-1)t}}{t^2 - 4} = \frac{t - 2t + 2}{t^2 - 4} = \frac{t(t-1)}{t^2 - 4}$
 $= \frac{-t+2}{t(t-1)} = \frac{-(t-2)}{t(t-1)} \cdot \frac{1}{(t-2)(t+2)} = \boxed{\frac{-1}{t(t-1)(t+2)}}$

7. $f(g(x)) = \frac{x^3+1}{x^3+2}$

Note: For problems like this one, try to find a repeating function that would "be plugged in".

$\hookrightarrow g(x) = x^3$ and $f(x) = \frac{x+1}{x+2} \Rightarrow f(g(x)) = \frac{x^3+1}{x^3+2}$ and $g(f(x)) = \left(\frac{x+1}{x+2}\right)^3$

8(a) Consider: $\frac{xy+y}{xz} = \frac{x(y+1)}{xz} = \frac{y+1}{z}$ } This is correct b/c the x can be factored out and cancelled.

$\frac{xy+y^2}{xz} \neq \frac{y+y^2}{z}$ } This is incorrect b/c we can't factor out an x in $xy+y^2$. So we cancel the x in the denominator.

(b) Consider $\sqrt{x^2 y^4} = \sqrt{(xy^2)^2} = xy^2$ } This is correct b/c there are 2 copies of xy^2 in $x^2 y^4$, so we can simplify the radical.
Note: $\sqrt{a^{2n}} = \sqrt{(a^n)^2} = a^n$

Consider $\sqrt{x^2 + y^4} \neq x + y^2$ } The mistake made here was simplifying each individual term. In order for this to be true, we need $(x+y^2)^2 = x^2 + y^4$ (what's under the radical).
 $\hookrightarrow (x+y^2)(x+y^2) = x^2 + 2xy^2 + y^4 \neq x^2 + y^4$

NOTE: $\sqrt{a^2 + b^2} \neq \sqrt{a^2} + \sqrt{b^2}$