

1. [15 points] Compute the following derivatives.

(a) $\frac{d}{dx} \left(\frac{x^2 + \pi^2}{x^3 + \sqrt{7}^3} \right)$. Do not simplify your answer.

(b) Let $f(u) = \frac{h(u)}{u^2 + 1}$, where $h(2) = -1$ and $h'(2) = 3$. Compute $f'(2)$.

(c) $\left((x^2 + 1)^3 (1 - 3x)^2 \right)'$. Do not simplify your answer.

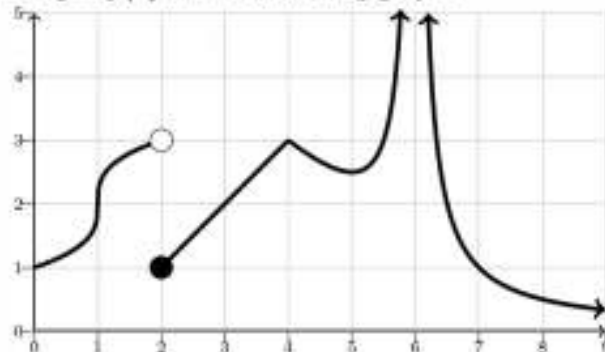
2. [14 points]

(a) State the limit definition of the derivative of a function $f(x)$.

(b) Compute $\frac{d}{dx}\left(\frac{1}{x^2+1}\right)$ using the limit definition of derivative.

3. [10 points] Compute the second derivative of $f(x) = \frac{x^2}{x+3}$ and simplify your answer.

4. [15 points] Suppose that $y = f(x)$ has the following graph:



- (a) For which numbers a does $f(x)$ fail to be continuous at a ? Give reasons using the definition of continuity?

- (b) For which numbers a does $f(x)$ fail to be differentiable at a ? Give reasons.

- (c) Find all x 's for which $f'(x) > 0$.

5. [16 points] We are adding trash to a brand new landfill. Assume that the amount of trash in the landfill at time t (= months since the landfill opened) is given by the formula

$$W(t) = 100t + 10t^2 \text{ tons of trash.}$$

- (a) How much trash was added to the landfill during the first six months of its operation?

- (b) Compute the rate of adding trash during this six month time period.

- (c) What was the rate of adding trash exactly six months after the landfill opened?

- (d) When you compare the answers to parts (b) and (c), what conclusion do you draw?

6. [10 points] Find the equation of the line tangent to the curve $y = \frac{x^2 + \sqrt{x} + 1}{2 - x}$ at the point where the x -coordinate is equal to 1.

7. [15 points] Let

$$f(x) = \frac{x^2}{\sqrt{x^2 - 1}}.$$

Note that $f(x)$ is defined when $x^2 > 1$, which holds when either $x > 1$ or $x < -1$.

- (a) Compute $f'(x)$ and simplify your answer as much as possible. Your final answer should be $f'(x) = \frac{x(x^2 - 2)}{(x^2 - 1)^{3/2}}$. To get full credit, I need to see every step of the simplification.

- (b) Find all points on the curve where the tangent line is horizontal.

8. [5 points] The production q of a company depends on both the capital investment K (in dollars) and the size of the labor force L (the number of workers). In economics, one frequently used formula for q in terms of K and L is the *Cobb-Douglas production function*

$$q = \sqrt{KL}.$$

Assuming the capital investment remains constant, compute the rate of change of production as the number of workers increases.

