



*Amherst College*  
*Department of Mathematics and Statistics*

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MATH 105

TEST #1

FALL 2015

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NAME: \_\_\_\_\_

**Read This First!**

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted. **Cell phones out of sight.**
- Please read each question carefully. Show **ALL** work clearly in the space provided. You may use the backs of pages for additional work space.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable
- Answers must be clearly labeled in the spaces provided after each question.
- The exam consists of Questions 1-6, which total to 100 points. Question 7 is a bonus question (5 points extra credit) that is optional.

**Grading - For Instructor Use Only**

Question:	1	2	3	4	5	6	7	Total
Points:	24	18	20	15	15	8	0	100
Score:								

1. [24 points] Compute the following limits. If  $+\infty$  or  $-\infty$  is a correct answer, please give it.

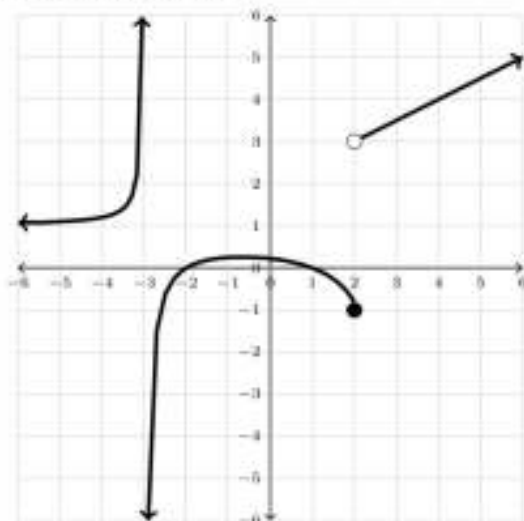
(a)  $\lim_{x \rightarrow 1} \frac{1+x^2}{1+x}$

(b)  $\lim_{x \rightarrow 1} \frac{x-1}{x^2+x-2}$

(c)  $\lim_{x \rightarrow 4^+} \frac{x^2-1}{4-x}$

(d)  $\lim_{x \rightarrow 1} \frac{\sqrt{x^2+1}-\sqrt{2}}{x-1}$

2. [18 points] Consider the following graph:



- (a) What is the domain of  $f$ ? Express your answer in interval notation.
- (b) For which  $x$ 's is  $f(x) = 0$ ?
- (c) For which  $x$ 's is  $f(x) < 0$ ? Express your answer in interval notation.
- (d) Is  $f$  continuous at 2? Explain your answer using the definition of continuity.

3. [20 points] Consider the function defined by

$$g(x) = \begin{cases} 1/x & x > 0 \\ 1 & x = 0 \\ 2 - x^2 & x < 0. \end{cases}$$

- (a) Draw the graph of  $g$ .

- (b) Use the graph of part (a) to find  $\lim_{x \rightarrow 0^+} g(x)$ ,  $\lim_{x \rightarrow 0^-} g(x)$ ,  $\lim_{x \rightarrow 0} g(x)$  and  $g(0)$ .

4. [15 points] Let

$$f(x) = \frac{x+1}{x+2} \quad \text{and} \quad g(x) = \frac{1-x}{1+x}.$$

Simplify  $f(g(x))$  as much as possible.

5. [15 points] Suppose we know the limits

$$\lim_{x \rightarrow 2} f(x) = 4, \quad \lim_{x \rightarrow 2} g(x) = 3, \quad \lim_{x \rightarrow 2} h(x) = 0.$$

- (a) What do the limit laws say about  $\lim_{x \rightarrow 2} \frac{f(x)}{g(x)}$ ?

- (b) What do the limit laws say about  $\lim_{x \rightarrow 2} \frac{h(x)}{g(x)}$ ?

- (c) What do the limit laws say about  ~~$\lim_{x \rightarrow 2} \frac{f(x) + h(x)}{h(x)}$~~   $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$ ?

6. [8 points] Find the equation of the line perpendicular to the line  $2x + 5y = 10$  that goes through the point  $(-\frac{1}{2}, 2)$ .

7. [5 points (bonus)] Let  $f(x) = 1 - x^2$ . Compute

$$\lim_{h \rightarrow 0} \frac{f\left(\frac{1}{x+h}\right) - f\left(\frac{1}{x}\right)}{h}.$$

