



Amherst College  
Department of Mathematics and Statistics

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MATH 105

MIDTERM 2

FALL 2018

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NAME: Solutions

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**Read This First!**

- Keep cell phones off and out of sight.
- Do not talk during the exam.
- You are allowed one page of notes, front and back. No other books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Please read each question carefully. Show **ALL** work clearly in the space provided. There is an extra page at the back for additional scratchwork.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable.

**Grading - For Instructor Use Only**

|           |   |    |   |    |   |   |       |
|-----------|---|----|---|----|---|---|-------|
| Question: | 1 | 2  | 3 | 4  | 5 | 6 | Total |
| Points:   | 8 | 24 | 8 | 12 | 8 | 0 | 60    |
| Score:    |   |    |   |    |   |   |       |

1. [8 points] Consider the function

$$f(x) = 2x^3 + 5x.$$

Find an equation for the tangent line to the graph  $y = f(x)$  at the point where  $x = -1$ .

$$\begin{aligned} f'(x) &= 2 \cdot 3x^2 + 5 \cdot 1 \\ &= 6x^2 + 5 \\ f'(-1) &= 6(-1)^2 + 5 \\ &= 11. \end{aligned}$$

We want the line through  $(-1, f(-1)) = (-1, 2(-1)^3 + 5(-1))$   
 $= (-1, -7)$

w/ slope  $f'(-1) = 11$ ,

ie.

$$\begin{aligned} (y+7) &= 11(x+1) \\ &= 11x+11 \end{aligned}$$

$$\Leftrightarrow \boxed{y = 11x + 4}$$

2. [24 points] Evaluate the derivative of each function. You do not need to simplify your answers.

(a)  $\frac{2x+1}{3x+1}$

$$f'(x) = \frac{2 \cdot (3x+1) - (2x+1) \cdot 3}{(3x+1)^2}$$

(b)  $\frac{2x^2 + 3\sqrt{x} + 5}{\sqrt{x}} = 2x^{3/2} + 3 + 5x^{-1/2}$

$$f'(x) = 2 \cdot \frac{3}{2} x^{1/2} + 0 + 5 \cdot \left(-\frac{1}{2}\right) x^{-3/2}$$

$$= \boxed{3x^{1/2} - \frac{5}{2}x^{-3/2}} \quad \left(\text{or } 3\sqrt{x} - \frac{5}{2\sqrt{x}}\right)$$

(continued on reverse)

(c)  $\sqrt{3 + (1+x)^4}$

$$f'(x) = \frac{1}{2\sqrt{3+(1+x)^4}} \cdot \frac{d}{dx} (3 + (1+x)^4)$$

$$= \frac{1}{2\sqrt{3+(1+x)^4}} \cdot 4(1+x)^3 \cdot \frac{d}{dx} (1+x) \rightarrow 1$$

(d)  $\frac{(2+3x)^2}{\sqrt{3-x}}$

$$f'(x) = \frac{2(2+3x) \cdot \frac{d}{dx} (2+3x) \cdot \sqrt{3-x} - (2+3x)^2 \cdot \frac{1}{2\sqrt{3-x}} \cdot \frac{d}{dx} (3-x)}{(\sqrt{3-x})^2}$$

$$= \frac{2(2+3x) \cdot 3 \cdot \sqrt{3-x} - (2+3x)^2 \cdot \frac{1}{2\sqrt{3-x}} \cdot (-1)}{3-x}$$

3. [8 points] Let

$$f(x) = \sqrt{1+x^2}.$$

Use the limit definition of the derivative to find  $f'(x)$ .

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\sqrt{1+(x+h)^2} - \sqrt{1+x^2}}{h} \cdot \frac{\sqrt{1+(x+h)^2} + \sqrt{1+x^2}}{\sqrt{1+(x+h)^2} + \sqrt{1+x^2}} \\ &= \lim_{h \rightarrow 0} \frac{(1+(x+h)^2) - (1+x^2)}{h \cdot [\sqrt{1+(x+h)^2} + \sqrt{1+x^2}]} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 1 - x^2}{h \cdot [\sqrt{1+(x+h)^2} + \sqrt{1+x^2}]} \\ &= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h \cdot [\sqrt{1+(x+h)^2} + \sqrt{1+x^2}]} \\ &= \frac{2x+0}{\sqrt{1+(x+0)^2} + \sqrt{1+x^2}} = \frac{2x}{2\sqrt{1+x^2}} \\ &= \boxed{\frac{x}{\sqrt{1+x^2}}} \end{aligned}$$

4. [12 points] Consider the function

$$f(x) = \frac{x^2 + 1}{2x + 1}.$$

- (a) Compute and simplify the derivative
- $f'(x)$
- .

$$\begin{aligned} f'(x) &= \frac{2x(2x+1) - (x^2+1) \cdot 2}{(2x+1)^2} \\ &= \frac{4x^2 + 2x - 2x^2 - 2}{(2x+1)^2} \\ &= \frac{2x^2 + 2x - 2}{(2x+1)^2} \\ &= \boxed{2 \cdot \frac{x^2 + x - 1}{(2x+1)^2}} \end{aligned}$$

- (b) Compute and simplify the second derivative
- $f''(x)$
- . Your final answer should be
- $\frac{10}{(2x+1)^3}$
- .
- 
- For full credit, show each step of your simplification.

$$\begin{aligned} f''(x) &= 2 \cdot \frac{(2x+1)(2x+1)^2 - (x^2+x-1) \cdot 2(2x+1) \cdot 2}{[(2x+1)^2]^2} \\ &= 2(2x+1) \cdot \frac{(2x+1)^2 - 4(x^2+x-1)}{(2x+1)^4} \\ &= 2 \cdot \frac{4x^2 + 4x + 1 - 4x^2 - 4x + 4}{(2x+1)^3} \\ &= 2 \cdot \frac{5}{(2x+1)^3} = \boxed{\frac{10}{(2x+1)^3}} \end{aligned}$$

5. [8 points] At what points is the tangent line to the graph  $y = (x+1)^2(2x-1)^3$  horizontal? For this problem, it is enough to state the  $x$ -coordinate only in your answer.

$$\begin{aligned} \frac{dy}{dx} &= 2(x+1) \cdot 1 \cdot (2x-1)^3 + (x+1)^2 \cdot 3(2x-1)^2 \cdot 2 \\ &= 2(x+1)(2x-1)^3 + 6(x+1)^2(2x-1)^2 \\ &= 2(x+1)(2x-1)^2 \cdot [(2x-1) + 3(x+1)] \\ &= 2(x+1)(2x-1)^2 \cdot [5x+2] \end{aligned}$$

$$\text{So } \frac{dy}{dx} = 0 \quad (\Leftrightarrow) \quad \boxed{x = -1, \frac{1}{2}, \text{ or } -\frac{2}{5}}$$

$$\begin{cases} x+1=0 & (\Leftrightarrow) x = -1 \\ 2x-1=0 & (\Leftrightarrow) 2x = 1 \\ & (\Leftrightarrow) x = 1/2 \\ 5x+2=0 & (\Leftrightarrow) 5x = -2 \\ & (\Leftrightarrow) x = -2/5. \end{cases}$$

6. [3 points (bonus)] Evaluate and simplify

$$\frac{d}{dx} \sqrt{1 + (5 + \sqrt{x/6})^{12}}$$

$$= \frac{1}{2\sqrt{1 + (5 + \sqrt{x/6})^{12}}} \cdot \frac{d}{dx} \left( 1 + (5 + \sqrt{x/6})^{12} \right)$$

$$= \frac{1}{2\sqrt{1 + (5 + \sqrt{x/6})^{12}}} \cdot 12(5 + \sqrt{x/6})^{11} \cdot \frac{d}{dx} (5 + \sqrt{x/6})$$

$$= \frac{1}{2\sqrt{1 + (5 + \sqrt{x/6})^{12}}} \cdot 12(5 + \sqrt{x/6})^{11} \cdot \frac{1}{2\sqrt{x/6}} \cdot \frac{d}{dx} (x/6)$$

$$= \frac{1}{2\sqrt{1 + (5 + \sqrt{x/6})^{12}}} \cdot \cancel{2}(5 + \sqrt{x/6})^{11} \cdot \frac{1}{\cancel{2}\sqrt{x/6}} \cdot \frac{1}{6}$$

$$= \boxed{\frac{(5 + \sqrt{x/6})^{11}}{2\sqrt{1 + (5 + \sqrt{x/6})^{12}} \cdot \sqrt{x/6}}}$$